Exchange Rate Volatility and Productivity Growth: the Role of Liability Dollarization
TECHNICAL APPENDIX

The log-linearized, reduced-form model

Let x_t denote the deviation from the non-stochastic steady state of X_t: $x_t = \frac{X_t - X}{X} \backsimeq \ln(X_t) - \ln(X)$.

We are interested in the behavior of π (time subscripts are dropped for simplicity). We thus log-linearize (6) and use the labor demand (5) to infer:

$$\pi = \underbrace{(1-\kappa)(\eta+1)y^T + \kappa(p^N - p^T + y^N)}_{\text{Gross profit effect}} + \underbrace{(1-\kappa)\eta(1-\alpha)p^T}_{\text{Debt valuation effect}}$$

where $\kappa = \frac{\frac{P^{N2}}{2P^T W}}{1-(1+r^*)d + \frac{P^{N2}}{2P^T W}}$ denotes the steady-state share of nontradables in the cash flows and $\eta = \frac{(1+r^*)d}{1-(1+r^*)d}$ denotes the steady-state ratio of debt repayments over the tradable consumption (tradable profit minus debt repayments). We have $0 < \kappa < 1$ and $\eta > 0$. The first and second terms of π represent respectively the tradable and nontradable gross profits valued in terms of tradables (or dollars). The last term represents the effect of the debt currency composition on the financing capacities of firms. For example, everything equal, a nominal exchange rate depreciation (appreciation), that is a rise in p^T (a fall) leads to a depreciation (appreciation) in the value of the nontradable gross profits, but it also alleviates (increases) the peso-denominated part of the debt when $\alpha < 1$. If $\alpha = 1$, debt repayments in terms of tradables are immune to nominal exchange rate variations and cannot hedge the variations in the tradable value of profits. However, one needs to consider how y^T, y^N, p^T and p^N vary jointly. To know how π reacts to the productivity shock u, it is then sufficient to know the behavior of production and prices, which we can derive from the following reduced-form model.

The log-linearization of the relative demand for tradables and nontradables (10) ($p^N - p^T = \frac{1}{\theta}(c^T - c^N)$) and the equilibrium conditions (8) ($c^N = y^N$) and (9) ($c^T = (\eta+1)y^T + \eta(1-\alpha)p^T$) gives:

$$p^N - p^T = \frac{1}{\theta}[(\eta+1)y^T + \eta(1-\alpha)p^T - y^N] \tag{15}$$

The relative price of nontradables in terms of tradables has to fall either if the production of nontradables rises or if the production of tradables falls. This also happens if $\alpha < 1$ and the nominal exchange rate appreciates (p^T falls), because this makes the peso-denominated debt increase which leaves less tradable goods to consume for the household.

Besides, the log-linearization of supply of nontradables (4) ($y^N = \frac{l}{2}$) and the labor demand (5) ($p^N + y^N = l$) yields:

$$y^N = p^N \tag{16}$$

Here we see that a deflation in p^N has a contractionary effect on y^N. This is because nominal wages are preset. As a consequence, a deflation in p^N depresses the production of nontradables through the rise of the real wage.

Moreover, by log-linearizing the supply for tradables (3), we obtain:

$$y^T = u \tag{17}$$

Finally, the two possible policy choices are the following:

- Flexible exchange rate:

$$p = 0$$

Besides, according to (11) ($p = \gamma p^T + (1-\gamma)p^N$) the flexible rule reduces to:

$$p^T = \frac{-(1-\gamma)}{\gamma} p^N \tag{18}$$

- Fixed exchange rate:

$$p^T = 0 \tag{19}$$

With only (15), (16), (17) and one of the two monetary rules (18) or (19), π can be inferred.

Reactions of quantities and prices to shocks

The reduced form model composed of (15), (16), (17) and one of the two monetary rules (18) or (19) is solved to obtain the following Lemma:

Lemma 1

- *Under a flexible exchange rate,*

$$p^{Nflex} = \frac{\gamma(\eta+1)u}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)}, \qquad p^{Tflex} = \frac{(1-\gamma)(\eta+1)u}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)}$$

$$p^{Nflex} - p^{Tflex} = \frac{(\eta+1)u}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)}$$

$$y^{Nflex} = \frac{\gamma(\eta+1)u}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)}, \qquad y^{Tflex} = u$$

- *Under a fixed exchange rate,*

$$p^{Nfix} = \frac{(\eta+1)u}{\theta+1}, \qquad p^{Tfix} = 0, \qquad p^{Nfix} - p^{Tfix} = \frac{(\eta+1)u}{\theta+1}$$

$$y^{Nfix} = \frac{(\eta+1)u}{\theta+1}, \qquad y^{Tfix} = u$$

Lemma 1 is used to establish Proposition 1:

Proof of Proposition 1

(i) From Lemma 1, if $u < 0$:

$y^{Nflex} > y^{Nfix} \Leftrightarrow \gamma(\theta+1) < \theta+\gamma+(1-\gamma)\eta(1-\alpha) \Leftrightarrow (1-\gamma)[\theta+\eta(1-\alpha)] > 0$: always true.

$p^{Nflex} - p^{Tflex} < p^{Nfix} - p^{Tfix} \Leftrightarrow \theta+1 > \theta+\gamma+(1-\gamma)\eta(1-\alpha) \Leftrightarrow \alpha > 1-\frac{1}{\eta}$, true for $\alpha = 1$.

(ii) From Lemma 1, we derive:

$$y^{Nflex} + p^{Nflex} - p^{Tflex} = \frac{\kappa(1+\gamma)(\eta+1)u}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)} < y^{Nfix} + p^{Nfix} - p^{Tfix} = \frac{2\kappa(\eta+1)u}{\theta+1}$$

if $u < 0$:

$$\Leftrightarrow \frac{(\kappa(1+\gamma)(\eta+1)}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)} > \frac{2\kappa(\eta+1)}{\theta+1}$$

$$\Leftrightarrow \kappa(1+\gamma)(\theta+1) > 2\kappa[\theta+\gamma+(1-\gamma)\eta(1-\alpha)]$$

after rearranging:

$$\Leftrightarrow \alpha > 1-\frac{\kappa(1-\theta)}{\eta}$$

is true for $\alpha = 1$ since $\theta < 1$

(iii) From Lemma 1, y^{Nflex} and $p^{Nflex} - p^{Tflex}$ are both decreasing in $(1-\alpha)$.. Therefore, $y^{Nflex} + p^{Nflex} - p^{Tflex}$ is decreasing in $(1-\alpha)$.

Proof of Proposition 2

From Lemma 1, we derive:

$$\pi^{flex}(u) = \frac{[\theta+\gamma+\kappa(1-\theta)](\eta+1)}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)}u$$

$$\pi^{fix}(u) = \frac{[\theta+1+\kappa(1-\theta)](\eta+1)}{\theta+1}u$$

$\Pi^i = E(\Pi) + \pi^i$, $i = \{flex, fix\}$, so Π^{flex} and Π^{fix} are of the same form as (2), with the following resulting aggregate shocks:

$$\sigma^{\pi flex} = \frac{[\theta+\gamma+\kappa(1-\theta)](\eta+1)}{\theta+\gamma+(1-\gamma)\eta(1-\alpha)}\sigma$$

$$\sigma^{\pi fix} = \frac{[\theta+1+\kappa(1-\theta)](\eta+1)}{\theta+1}\sigma$$

(i) According to section 2.1.1, a fixed exchange rate yields higher growth than a flexible one (that is, $E(\rho^{fix}) > E(\rho^{flex})$), if and only if $\sigma^{\pi flex} > \sigma^{\pi fix}$.

$\sigma^{\pi flex} > \sigma^{\pi fix} \Leftrightarrow [\theta+\gamma+\kappa(1-\theta)](\theta+1) > [\theta+1+\kappa(1-\theta)][\theta+\gamma+(1-\gamma)\eta(1-\alpha)]$

$\Leftrightarrow \alpha > 1-\frac{\kappa(1-\theta)}{\eta[1+\kappa+(1-\kappa)\theta]}$: true for $\alpha = 1$ since $\theta < 1$.

(ii) $\sigma^{\pi flex} - \sigma^{\pi fix}$ is a measure of the growth differential between the fixed and flexible exchange rate regimes.

We have $\frac{\partial(\sigma^{\pi flex} - \sigma^{\pi fix})}{\partial\alpha} = \frac{\partial\sigma^{\pi flex}}{\partial\alpha} > 0$, which means that the growth differential decreases when α diminishes.

(iii) $\sigma^{\pi fix} > \sigma^{\pi flex} \Leftrightarrow \alpha > 1 - \frac{\kappa(1-\theta)}{\eta[1+\kappa+(1-\kappa)\theta]}$ and $1 - \frac{\kappa(1-\theta)}{\eta[1+\kappa+(1-\kappa)\theta]} > 0 \Leftrightarrow \frac{\kappa(1-\theta)}{\eta[1+\kappa+(1-\kappa)\theta]} < 1$

This means that if the indebtment level η and the elasticity of substitution θ are high and if the share of nontradable production κ is low, then there exist a level of dollarization $\alpha > 0$ under which a peg is more growth-enhancing than a float.