Technical Appendix

1 Hand-to-mouth workers

We assume that workers have log-utility:

$$\sum_{s=0}^{\infty} \beta^s \ln(c^w_{t+s})$$

They supply inelastically one unit of labor paid at rate w and hold bonds B^w. They are thus subject to the following budget constraint:

$$c^w_t + B^w_{t+1} = w_t + r_t B^w_t$$

We also assume that they face a no-borrowing constraint:

$$B^w_{t+s} \geq 0$$

Proposition 1 *If $r_{t+s} \leq r^*$ for all $s \geq 0$, $w_t \leq w_{t+s}$ for all $s \geq 1$ and $B^w_t = 0$, then $c^w_t = w_t$ and $B^w_{t+1} = 0$.*

Proof. The intertemporal budget constraint of this problem is

$$\sum_{s=0}^{\infty} \frac{c^w_{t+s} - w_{t+s}}{\prod_{k=1}^{s} r_{t+s}} = 0$$

And the Euler equations are

$$c^w_{t+1+s} = \beta r_{t+1+s} c^w_{t+s} (1 + \lambda^w_{t+1+s})$$

for all $s \geq 0$, where λ^w is the multiplier of the no-borrowing constraint.

Suppose $B^w_{t+1} > 0$. Then, because $B^w_t = 0$, we have $c^w_t < w_t$. The intertemporal budget constraint implies:

$$c^w_t - w_t = -\sum_{s=1}^{\infty} \frac{c^w_{t+s} - w_{t+s}}{\prod_{k=1}^{s} r_{t+s}}$$

$c^w_t < w_t$ therefore implies that there exists at least one $s_0 \geq 1$ such that $c^w_{t+s_0} > w_{t+s_0}$. Since $w_t \leq w_{t+s}$ for all $s \geq 1$ by hypothesis and $c^w_t < w_t$, then we have $c^w_{t+s_0} > c^w_t$.

Consider the first s_0 such that $c^w_{t+s_0} > w_{t+s_0}$. For all $0 \leq s < s_0$, we have $c^w_{t+s} \leq w_{t+s}$. The instantaneous budget constraints then imply $B^w_{t+1+s} \geq (\prod_{k=1}^{s} r_{t+k})B^w_{t+1} > 0$, which means that $\lambda^w_{t+1+s} = 0$ for $0 \leq s < s_0$. The Euler equations then imply:

$$c^w_{t+s_0} = \beta^{s_0}(\prod_{k=1}^{s_0} r_{t+k})c^w_t$$

Since $r_{t+s} \leq r^* = 1/\beta$ for all $s \geq 0$, then $c^w_t \geq c^w_{t+s_0}$, which constitutes a contradiction.

Therefore, we must have $B^w_{t+1} = 0$ and thus $c^w_t = w_t$. ∎

In the model, the wage is given in equilibrium by equation (??) with $l_t = 1$: $w_{t+1} = \frac{1-\alpha}{\alpha} r_{t+1}K_{t+1}$. In a small open economy, we have $r_{t+1} = r^*$. When it experiences a growth or convergence episode, K_{t+1} is growing over time, and so is w_{t+1}. According to Proposition 1, the hand-to-mouth behavior of workers is a natural outcome of the growth in wages in these cases (assuming that the initial B^w is equal to zero).

In the two-country case, we check ex-post that the wage is indeed growing in the Emerging country and that $r_t \leq r^*$ in the experiment that we consider, that is, growth in the developed country, so the hand-to-mouth behavior can also be an equilibrium consequence of growth. However, in the Developed country, the wage increases at first and then decreases, so Proposition 1 does not apply.

To check whether our results do not depend on the assumption of hand-to-mouth workers in the Developed country, we simulate the two-country model when workers in the Developed country behave optimally and do not face any borrowing constraint. Because they anticipate a temporary growth in their labor income, the workers borrow as they want to smooth their consumption. This constitutes a supply of bonds that potentially crowds out investment by firms in the Developed country. However, we find that investment is only partially crowded out and the Developed country still experiences an investment boom as a response to the increased liquidity demand by the Emerging country, although the investment boom is smaller than with hand-to-mouth workers. Investment and bond holdings in the Emerging country are barely affected by this hypothesis.

In contrast, when we assume that workers do not face borrowing constraints in the Emerging country, our results are overturned. The Emerging country experiences capital inflows as the supply of bonds by domestic workers exceeds the liquidity demand by domestic firms. Indeed, because workers in the Emerging country anticipate a permanent increase in their wages, their supply of bonds is larger than that of workers in the Developed country.

2 FDI

We consider here an extension of the benchmark case that accounts for foreign direct investment (FDI). We use this extension to determine whether our model can replicate the phenomenon of two-way flows described in the literature (inflows of FDI along with outflows of bonds) and whether the net outflows from developing countries are robust to the introduction of alternative sources of financing that are not subject to credit frictions.

In order to achieve that, one assumption is introduced: foreign investors have the possibility to operate projects domestically. We take the point of view of an emerging countries that faces a constant world interest rate $r^* = 1/\beta^*$. We assume that $\beta < \beta^*$, so the economy is constrained in the steady state.

2.1 No FDI costs

A foreign investor has the possibility to operate local projects in the domestic country. We denote respectively by K^F and l^F the total capital invested by foreign investors in the domestic country and the total labor force they hire in the local market. Assume first that there are no costs at operating domestic projects for the foreign investors

$$\pi(K_{t+1}^F, l_{t+1}^F) = A_{t+1}(K_{t+1}^F)^\alpha (l_{t+1}^F)^{1-\alpha} - r^{*2} K_{t+1}^F - r^* w_t l_{t+1}^F$$

The first-order conditions are the following:

$$\alpha A_{t+1} \left(\frac{K_{t+1}^F}{l_{t+1}^F} \right)^{-(1-\alpha)} = r^{*2}$$

$$(1-\alpha) A_{t+1} \left(\frac{K_{t+1}^F}{l_{t+1}^F} \right)^{\alpha} = r^* w_{t+1}$$

This defines the domestic wage w_{t+1} as the first best one:

$$\frac{w_{t+1}}{A_{t+1}} = \tilde{w}_{t+1} = \hat{w}$$

As a result, domestic firms are not constrained anymore. Indeed, for this wage, the level of production is undetermined. They adjust the scale of production in order to avoid being constrained. The rest of the labor force is hired by foreign investors.

In that case, both the level of domestic capital and the amount of FDI are undetermined. Besides, since domestic firms are unconstrained, their wealth $S = B + K$ decreases and eventually converges to zero:

$$\frac{S_{t+1}}{S_{t-1}} = (\beta r^*)^2 < 1$$

This is because domestic agents are impatient. When they are constrained, this impatience is compensated by the liquidity premium on capital. But when they are unconstrained, as it is the case here, they consume more than the return on their portfolio. As a result, domestic agents are completely crowded out by foreigners in the long-run.

2.2 Iceberg costs

We assume now that, when investing abroad, foreign investors face an iceberg cost τ, with $0 < \tau \leq 1$. This cost can be viewed as stemming from informational or administrative issues, or from expropriation risk. The intertemporal profits the entrepreneurs get from each projects are then:

$$\pi(K_{t+1}^F, l_{t+1}^F) = (1-\tau)A_{t+1}(K_{t+1}^F)^\alpha (l_{t+1}^F)^{1-\alpha} - r^{*2} K_{t+1}^F - r^* w_{t+1} l_{t+1}^F$$

The first-order conditions are the following:

$$(1-\tau)\alpha A_{t+1}\left(\frac{K_{t+1}^F}{l_{t+1}^F}\right)^{-(1-\alpha)} = r^{*2}$$

$$(1-\tau)(1-\alpha)A_{t+1}\left(\frac{K_{t+1}^F}{l_{t+1}^F}\right)^{\alpha} = r^* w_{t+1}$$

These equations characterize the wage offered by foreign investors as the first-best wage with a wedge:

$$w_{t+1} = (1-\tau)^{\frac{1}{1-\alpha}}\hat{w} \qquad (1)$$

For foreigners to be willing to establish firms in the domestic country, we must have$(1-\tau)^{\frac{1}{1-\alpha}}\hat{w}$ higher than $(1-\alpha)r^*K_{t+1}/\alpha$, the equilibrium wage that prevails in the absence of FDI. Otherwise, no domestic agent would like to work for the foreign firm. The condition for FDI to take place is therefore the following:

$$K_t \leq (1-\tau)^{\frac{1}{1-\alpha}}\hat{K} \tag{2}$$

where $\hat{K} = (A\alpha\beta^{*2})^{\frac{1}{1-\alpha}}$is the level of capital that would prevail in the domestic country in the absence of credit constraints. This condition also implies $l_t \leq 1$. $1-l_t$ is then the amount of labor hired by foreign firms.

Suppose now that condition (2) is fulfilled, and that foreigners enter the market. We now consider the dynamics of the economy following the establishment of FDI. The market wage is given by (1). Since $w_{t+1} < \hat{w}$, the domestic entrepreneurs are still constrained. The dynamic equation for capital is still valid, but domestic labor is now defined by:

$$l_t = \frac{(1-\alpha)r^*}{\alpha(1-\tau)^{\frac{1}{1-\alpha}}\hat{w}}K_t$$

The evolution of domestic capital K is then given by:

$$K_{t+1} = \frac{A_{t-1}}{A_{t+1}}\frac{(\beta r^*)^2}{1-\tau}K_{t-1}$$

If $A_{t-1} = A_{t+1}$, the dynamics of capital and total wealth S (since B is proportional to K) follows:

$$\frac{K_{t+1}}{K_{t-1}} = \frac{S_{t+1}}{S_{t-1}} = \frac{(\beta r^*)^2}{1-\tau} = \frac{(\bar{K}/\hat{K})^{1-\alpha}}{(1-\tau)}$$

where $\bar{K} = (A\alpha\beta^2)^{\frac{1}{1-\alpha}}$ is the steady-state of the domestic economy without FDI. The dynamics of K and S depend on τ.

If $\bar{K} < (1-\tau)^{\frac{1}{1-\alpha}}\hat{K}$, then K and S converge to zero. Indeed, when the impediments on FDI are small, the dynamics are similar to the case with $\tau = 0$. $\bar{K} > (1-\tau)^{\frac{1}{1-\alpha}}\hat{K}$ If $\bar{K} > (1-\tau)^{\frac{1}{1-\alpha}}\hat{K}$, then K increases until $K_t \geq (1-\tau)^{\frac{1}{1-\alpha}}\hat{K}$, where FDI are completely crowded out ($l_t = 1$). K converges then towards $\bar{K}$ as in the benchmark model. In this case, the costs are too large to sustain FDI in a permanent fashion. In the knife-edge case where $\bar{K} = (1-\tau)^{\frac{1}{1-\alpha}}\hat{K}$, the economy stays stationary after the initial establishment of FDI.

2.3 Unemployment in the domestic economy and increasing iceberg costs

In the two preceding examples, FDI and domestic capital move in opposite directions. Therefore, even when firms are still constrained (when $\tau > 0$), FDI and bonds are substitutes. This pertains mainly to the fact that there is full employment in the domestic economy, so, if we abstract from the movements in capital/labor ratio, one unit of FDI crowds out one unit of domestic capital. FDI can enter only if they offer a higher wage, and this higher wage crowds out domestic firms in the labor market.

However, in emerging countries, we observe that foreign firms often establish in the domestic country without affecting the domestic wage, sometimes for a long period. We need two more ingredients to introduce this feature in our model: (i) domestic firms are not able to hire all the domestic labor force so a fraction of this labor force is unemployed and the domestic wage is equal to the reservation wage $\underline{w}$; (ii) total FDI labor l^F has negative externalities on the iceberg cost, so FDI are not able to hire all the remaining labor force and the domestic wage remains equal to $\underline{w}$, even after the country opens to FDI.

(i) corresponds to the labor market situation described by equilibrium (0) in Figure 1. In order to represent (ii), we assume that the iceberg cost depends on the aggregate labor l^F hired by foreigners and that foreign investors are atomistic, so they do not internalize this cost. More specifically, $\tau = \tau(l^F)$ with $\tau(0) = 0$ and $\tau' > 0$.

$$w_{t+1} = (1 - \tau(l^F_{t+1}))^{\frac{1}{1-\alpha}} \hat{w}$$

This defines the labor demand by foreign firms as:

$$l^F(w_{t+1}) = \tau^{-1} \left[1 - \left(\frac{w_{t+1}}{\hat{w}} \right)^{1-\alpha} \right]$$

Similarly, we can write the labor demand by domestic firms as:

$$l(w_{t+1}, K_{t+1}) = \frac{(1-\alpha) r^*}{\alpha w_{t+1}} K_{t+1}$$

where K_{t+1} is independently defined by current wealth W_t.

Finally, to completely characterize the states where (ii) apply, we assume that, when the country opens to FDI, domestic and foreign firms cannot hire the total available labor force:

$$l^F(\underline{w}) + l(\underline{w}, K_{t+1}) < 1$$

which gives a condition on $\underline{w}$ and W_t.

When the country opens to FDI, foreign firms invest $K^F(\underline{w}) = \frac{\alpha \underline{w} l^F(\underline{w})}{(1-\alpha)r^*}$ without affecting whatsoever the domestic dynamics. Indeed, in this transitory state with unemployment, labor supply is perfectly elastic, so foreign investors do not crowd out domestic firms. The dynamics of domestic capital follows:

$$K_{t+1} = \alpha\beta^2 A_{t-1} \left(\frac{(1-\alpha)r^*}{\alpha \underline{w}} \right)^{1-\alpha} K_{t-1}$$

It can be shown that, during this phase, $K_{t+1} > K_{t-1}$ if and only if $\underline{w} < \bar{w}$, where $\bar{w} = \frac{(1-\alpha)r^* \bar{K}}{\alpha}$. This means that the reservation wage should be lower than the steady-state wage in the absence of FDI, which is a reasonable assumption. As a result, during this phase, the domestic country exhibits positive FDI, capital accumulation and, since bonds are proportional to capital, a growing liquidity demand.

Figure 1: Labor market equilibrium with a reservation wage