

Booms and Busts with dispersed information

ONLINE APPENDIX

Not for publication

Contents

A Extensions	2
A.1 Alternative shocks	2
A.2 Alternative information structure	5
A.3 Credit and endogenous initial signal	8
A.4 More dynamics	10
B Model with constant terms	13
B.1 The reduced-form model	14
B.2 The solution	14
C Main proofs	15
C.1 Preliminaries	15
C.2 Derivation of Equations (21) and (22)	16
C.3 Derivation of Equation (23)	16
C.4 Derivation of Equations (24) and (25)	17
C.5 Proof of Corollary 1	18
C.6 Proof of Proposition 3	18
C.7 Proof of Proposition A.1	18
D Additional proofs	19
D.1 Proof of Proposition 3 (full derivations)	19
D.2 Additional public signal	21
D.3 Additional private signal	22
D.4 Proof of Proposition A.1 (full derivations)	23
E Numerical simulations	24
F Additional numerical results	28

A Extensions

We address several limitations of the benchmark model. First, we discuss which other categories of shocks can generate booms and busts. We then examine alternative information structures. One extension shows how the model can be consistent with credit-fuelled booms and busts. Lastly, we show how our mechanism extends to a more dynamic model. The results are summarized here, but more details can be found in the online appendix. Except for the dynamic extension, these analyses are conducted within the simple version of the model where Assumptions 1 and 2 hold.

A.1 Alternative shocks

In this section we extend the model to accommodate other shocks. We first extend the model to incorporate a final good sector to be able to incorporate productivity shocks in the final good sector. We demonstrate that noisy signals about the final good productivity can generate booms and busts originating from the intermediate good sector. By contrast, productivity shocks in the intermediate good sector does not generate booms and busts. Booms and busts can thus arise from optimism about productivity but only if it concerns downstream production chains. We then consider another category of demand shocks: shocks to government spending, and show that noisy signals on government spending can generate booms and busts.

In order to make our analysis clearer, we focus here on the simple version of the model, where Assumptions 1 and 2 are satisfied.

Shocks to the productivity of the intermediate good sector We assume that the production of the differentiated goods is affected by productivity shocks A , so that the production function is modified as follows:

$$Q_{it} = AL_{it}$$

We assume that the firm cannot observe $a = \log(A)$ directly. Instead, firms observe a noisy signal of a , $a_i = a + \theta + \lambda_i$, and observe their mark-up at the end of period 2. One can think of imperfect information within the firm, where the manager receives noisy signals about the firm's productivity.¹

¹This is not an unrealistic assumption, as marginal costs are notoriously difficult to measure (as opposed to the average cost).

The manager of firm i sets quantities in order to satisfy:

$$E_i(p_i - w) = -E_i(a)$$

To illustrate the effect of noise shocks on productivity, we consider the simple case with $\eta = 0$. Using the individual and aggregate demand equations, the optimal individual supply can be written as a function of expected aggregate supply and expected shocks:

$$\hat{q}_i = (1 + \kappa_a)E_i(a) - \kappa_a E_i(\hat{q}) \quad (28)$$

The relevant information sets here are $\Omega_{i1} = \{a_i\}$ and $\Omega_{i1} = \{a_i, p_{i1} - w_1\}$.

In the first period, the firm's problem with productivity shocks is strictly similar to the firm's problem with demand shocks solved in Section 2. We can therefore easily derive the following:

$$\hat{q}_{i1} = K_a a_i \quad (29)$$

with $K_a = (1 + \kappa_a)\sigma_a^2 / [(1 + \kappa_a)(\sigma_a^2 + \sigma_\theta^2) + \sigma_\lambda^2]$. As a consequence, at the aggregate level, firms produce the following quantities:

$$\hat{q}_1 = K_a(a + v) \quad (30)$$

Because firms do not observe productivity, the new signal received by firms is not their mark-up $p_i - w + a$, but their real price $p_i - w$, which they can filter from the influence of their own supply:

$$p_{i1} - w_1 + \frac{1}{\tilde{\sigma}}\hat{q}_{i1} = -\frac{\tilde{\sigma} - 1}{\tilde{\sigma}}\hat{q}_1 = -\frac{\tilde{\sigma} - 1}{\tilde{\sigma}}K_a(a + v)$$

Importantly, in the case of productivity shocks, both the fundamental and the noise shocks are supply shocks, so they both affect the real price negatively, as $K_a > 0$. As a consequence, contrary to demand shocks, a positive noise shock about productivity does not generate a negative signal on the fundamental through prices. It thus does not generate boom-bust cycles.

Shocks to the productivity of the final good sector Here, we introduce a final good sector that produces the final consumption good by using intermediate inputs. The period- t utility is modified:

$$U_t = \Psi \frac{(Q_t^c)^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\eta}}{1+\eta}$$

where Q_t^c is the household consumption of the final good. The final good is produced and sold competitively by a representative firm with the following production function:

$$Q_t^c = AQ_t$$

where A is the productivity of the final good sector and Q is defined as before, except that it is not the consumption bundle but a partial production function that aggregates intermediate inputs. The intermediate inputs are produced by firms as described in the benchmark model.

We assume that there is no preference shock ($\Psi = 1$), but there are instead shocks to $a = \log(A)$ that have mean zero and standard error σ_a . a is realized at the beginning of period 1, and is not observed by firms in the intermediate goods sector. Instead, firms observe a noisy signal of a , $a_i = a + \theta + \lambda_i$, and observe their mark-up at the end of period 2.

Since the final good sector is competitive, we have $P^c = P/A$, where P is, as before, the aggregate price of the intermediate inputs while P^c is the price of the final good. We thus have

$$w_t - p_t = \sigma q_t - (1 - \gamma)a \tag{31}$$

where we used $w_t - p_t^c = \gamma q_t^c + \eta q_t$, $p_t^c = p_t - a$ and $q_t^c = q_t + a$.

The rest of the model is unaffected, except that ψ is replaced with $(1 - \gamma)a$. In the baseline model, ψ played the role of an unobserved negative shock to the real marginal production cost (the real wage). Here, when $\gamma < 1$, a plays the same role. As the final good producer is ready to pay a higher price p for intermediate goods, their real marginal production cost $w - p$ declines. Therefore, an aggregate noise shock about a generates the same type of dynamics as an aggregate noise shock about ψ .

Government spending shocks We consider the same model with the final good sector, and assume that there are neither preference shocks ($\Psi = 1$), nor productivity shocks ($A = 1$), but there are instead shocks to government spending. Suppose the government follows the rule: $G_t = \bar{g}Q_t e^\nu$, $t = 1, 2$, where G_t is the spending of the government in final goods, \bar{g} the average share of government spending in output and ν a permanent shock to the share of government spending in total output, realized at the beginning of period 1, and not observed by firms.

In logs, we have:

$$g_t = q_t + \nu + \log(\bar{g})$$

Government spending affects the budget constraint of the government: $M_t - M_{t-1} = T_t + G_t$. We assume that ν is not observed directly by firms, as before. Instead, firms observe a noisy signal $\nu_i = \nu + \theta + \lambda_i$ at the beginning of period 1 and $p_{i1} - w_1$ at the end of period 1.

Total output, in equilibrium, is equal to the sum of household consumption and government spending: $Q_t = Q_t^c + G_t$. This yields, in logs:

$$q_t = q_t^c + \frac{\bar{g}}{1 - \bar{g}}[\nu + \log(\bar{g})]$$

The labor supply equation (10) then becomes

$$w_t - p_t = \sigma q_t - \frac{\gamma \bar{g}}{1 - \bar{g}}[\nu + \log(\bar{g})] \quad (32)$$

The rest of the model is unaffected. In the baseline model, ψ played the role of an unobserved negative shock to the real wage. Here, ν plays the same role, because an increase in government spending makes goods relatively more expensive compared to leisure. Therefore, an aggregate noise shock about ν would generate the same type of dynamics as an aggregate noise shock about ψ .²

A.2 Alternative information structure

Here, we consider alternative information structures. First, in order to show the role played by dispersed information in our model, we assume that the exogenous signal on ψ is a public signal, so that firms all observe $\psi + \theta$. We then come back to our baseline information structure, but add additional signals, in order to evaluate whether boom-and-bust dynamics are robust to alternative information structures. This analysis is conducted within the simple version of the model where Assumptions 1 and 2 hold in order to make it more transparent.

Public exogenous signal Suppose that the exogenous signal observed by firms was public, so that firms, instead of observing ψ_i , observed $\psi_\theta = \psi + \theta$. As in our main model, we would have

$$\hat{q}_{i1} = K_\psi \psi_\theta$$

²Note that, in an open economy, foreign demand would play a role similar to g_t .

with $0 < K_\psi < 1$ (but with a different value than in the baseline). In that case, it is easy for firms to infer \hat{q}_1 , even though they do not observe it directly, as $\hat{q}_1 = K_\psi \psi_\theta$. Then, as $p_{i1} - w_1$ is a function of ψ , \hat{q}_1 and \hat{q}_{i1} , it would be straightforward to infer ψ by combining information on $p_{i1} - w_{1,q1}$ and \hat{q}_{i1} . It is therefore essential for the boom-and-bust mechanism that firms initially receive a *private* signal that has an *aggregate* error.

Of course this result is trivial since there are as many shocks to identify (ψ and θ) as signals ($p_{i1} - w_1$ and ψ_θ). However, even if $p_{i1} - w_1$ was observed with noise, we would still not have a boom-and-bust. For example, suppose that firms observe $p_{i1} - w_1 + z_i$, where z_i follows a normal distribution with mean zero and standard error σ_z . Firms can still use \hat{q}_1 to clean the mark-up from θ : $p_{i1} - w_1 + \frac{1}{\bar{\sigma}} \hat{q}_1 + \frac{\bar{\sigma}-1}{\bar{\sigma}} \hat{q}_1 = \psi + z_i$. In short, following a positive shock on θ , firms observe a decline in mark-up, but they can relate this decline to a high aggregate supply, so they do not confuse it with a depressed demand.

Adding a public signal We have established that firms must receive a *private* signal with an *aggregate* error for booms and busts to arise. What if firms received other private signals that do not have an aggregate error? Or received other signals with aggregate noise, but that are not private?

We consider first the case with a public noisy signal by supposing that, in addition to ψ_i , firms observed $\psi_e = \psi + e$ at the beginning of period 1, where e follows a normal distribution with mean zero and standard error σ_e .

We solve this case analytically and show that the main message of the model is not affected. In a nutshell, public signals make the forecasts of ψ more precise, and limit the impact of θ , but do not radically change the effect of θ . In particular, the structure of the filtered mark-up is unchanged and plays the same role as before.

The period-1 supply is affected as follows (see proof in the Online Appendix):

$$\hat{q}_{i1} = (1 - \alpha)K_\psi \psi_i + \alpha \psi_e \tag{33}$$

where K_ψ is defined as before, and $0 < \alpha < 1$, with $\alpha = \delta/(\sigma_e^2 + \delta)$, with δ a constant that is independent from σ_e .

At the aggregate level, firms produce the following quantities:

$$\hat{q}_1 = [(1 - \alpha)K_\psi + \alpha]\psi + (1 - \alpha)K_\psi \theta + \alpha e$$

As in the simple version of the model, firms under-react to the fundamental shock ψ and over-react to the noise shock θ . But because they receive an additional signal on ψ , they react more to ψ and less to θ , as $\alpha > 0$.

At the end of period 1, firms' mark-ups constitute a new signal. As Equation (18) is still valid, we can derive a filtered mark-up

$$p_{i1} - w_1 + \frac{1}{(1 + \kappa_a)} \hat{q}_{i1} + \frac{1}{(1 + \kappa_a)} \alpha \psi_e = \left(1 - \frac{\kappa_a}{(1 + \kappa_a)} (1 - \alpha) K_\psi \right) \psi - \frac{\kappa_a}{(1 + \kappa_a)} (1 - \alpha) K_\psi \theta$$

This represents the mark-up filtered from the influence of individual production \hat{q}_{i1} but also from the share of aggregate production that is explained by the public signal ψ_e , as it is common knowledge. Note that this filtered mark-up has the same structure as in the baseline model, up to the coefficient $1 - \alpha$, which depends on the precision of the public signal. When the signal becomes less precise (σ_e goes to infinity), α goes to zero and the filtered mark-up goes to its value in the baseline model. With a more precise signal, α is larger and the mark-up responds less to θ and more to ψ , but has essentially the same structure as in the baseline model. Therefore, shocks on θ still generate boom-and-bust dynamics.

Adding a private signal Suppose now that, in addition to ψ_i , firms received a purely private signal $\psi_{ui} = \psi + u_i$ at the beginning of period 1, where u_i follows a normal distribution with mean zero and standard error σ_u .

Again, we show that the main message of the model is not affected. The period-1 supply is affected as follows (see proof in the Online Appendix):

$$\hat{q}_{i1} = (1 - \alpha) K_\psi \psi_i + \alpha \psi_{ui} \tag{34}$$

where K_ψ is defined as before, and $0 < \alpha < 1$, with $\alpha = \delta / (\sigma_u^2 + \delta)$, with δ a constant that is independent from σ_u .

At the aggregate level, firms produce the following quantities:

$$\hat{q}_1 = [(1 - \alpha) K_\psi + \alpha] \psi + (1 - \alpha) K_\psi \theta$$

Again, firms under-react to the fundamental shock ψ and over-react to the noise shock θ . But because they receive an additional signal on ψ , they react more to ψ and less to θ , as $\alpha > 0$. Note that the difference with the public signal case is that the noise averages out to zero on the aggregate.

At the end of period 1, firms' mark-ups constitute a new signal. The filtered mark-up is now

$$p_{i1} - w_1 + \frac{1}{(1 + \kappa_a)} \hat{q}_{i1} = \left(1 - \frac{\kappa_a}{(1 + \kappa_a)} [(1 - \alpha)K_\psi + \alpha] \right) \psi - \frac{\kappa_a}{(1 + \kappa_a)} (1 - \alpha)K_\psi \theta$$

This represents the mark-up filtered from the influence of individual production \hat{q}_{i1} , but not from the contribution of the private signals ψ_{ui} , as they are not common knowledge. Note that this filtered mark-up has the same structure as in the baseline model, but depends on α . As in the case of public signals, when the signal becomes less precise (σ_u goes to infinity), α goes to zero and the filtered mark-up goes to its value in the baseline model. With a more precise signal, α is larger and the mark-up responds less to both θ and ψ , but has essentially the same structure as in the baseline model. Therefore, shocks on θ still generate boom-and-bust dynamics.

A.3 Credit and endogenous initial signal

We now introduce credit in order to account for the typical surge in credit that characterizes booms and busts. To do so, we introduce a non-produced traded good X , in fixed supply \bar{X} in the country but in infinite supply from the rest of the world. Households can exchange good X with the rest of the world and save or borrow vis-à-vis the rest of the world. We focus again on the simple version of the model with Assumptions 1 and 2 to better illustrate the mechanism. Strategic substitutability still affects the nontradable sector where firms produce differentiated goods as described in Section 1. In this extension, we also endogenize noise shocks by introducing an initial period 0, where temporary aggregate and idiosyncratic demand shocks can appear. These temporary demand shocks generate noise because firms cannot distinguish them from the permanent demand shock. Therefore, an aggregate temporary demand shock will generate credit among households and in the same time mislead firms about the true value of the permanent shock.

We introduce a demand for a traded good by amending the model of Section 1 in the following way. The specification of the utility (2) becomes for $t = 1, 2$:

$$U_t = \Psi \log (Q_t^\mu X_t^{1-\mu}) - L_t \tag{35}$$

with $0 < \mu < 1$. μ is the share of nontradable goods in consumption. When $\mu = 1$, the utility function boils down to (2) where $\gamma = 1$ and $\eta = 0$. Q now refers to the

consumption of non-traded goods. In the initial period $t = 0$, utility is now:

$$U_0 = \Psi\Theta \log(Q_t^\mu X_t^{1-\mu}) - L_t \quad (36)$$

with $Q_0 = \left(\int_0^1 \Lambda_i Q_i^{1-\rho} di\right)^{\frac{1}{1-\rho}}$. $\theta = \log(\Theta)$ is a temporary aggregate demand shock and $\lambda_i = \log(\Lambda_i)$ is a temporary idiosyncratic demand shock for good i , where θ and λ_i have the same characteristics as described in Section 1. The household now maximizes $U = U_0 + \beta U_1 + \beta^2 U_2$ subject to the budget constraints:

$$\int_0^1 P_{it} Q_{it} di + M_t + P_t^x X_t + r P_t^x D_{t-1} = W_t L_t + \int_0^1 \Pi_{it} di + M_{t-1} + T_t + P_t^x \bar{X} + P_t^x D_t$$

for $t = 0, 1, 2$. P_t^x is the price of good X in nominal terms and D_t is international borrowing in terms of tradable goods, which yields interest $r = 1/\beta$. Households now can trade intertemporally with the rest of the world through D . We assume that they start with no international debt so $D_{-1} = 0$.

The cash-in-advance constraint and the government budget constraint are the same as before, which yields (11).

The aggregate and individual demands remain as described in Equations (8) and (10) in periods 1 and 2, except that we have $\gamma = 1$ and $\eta = 0$. In period 0, they are additionally affected by the aggregate and individual transitory shocks θ and λ_i :

$$q_{i0} = q_0 - \frac{1}{\rho} [p_{i0} - p_0 - \lambda_{i0}]$$

$$w_0 - p_0 = q_0 - \psi - \theta$$

Ex ante, firms do not have any information, so they produce $q_i = q = 0$. As a result, in period 0, they observe mark-ups, which gives the initial signal ψ_i :

$$\psi_i = p_{i0} - w_0 = \psi + \theta + \lambda_i$$

Period-0 mark-ups are therefore an imperfect signal of the permanent demand shock ψ . This signal is perturbed by the aggregate and idiosyncratic demand shocks θ and λ_i . This gives an economic significance to the initial signal ψ_i in the baseline model.

We derive the following Proposition (see proof in the Appendix):

Proposition A.1 (Boom-busts and capital flows). *Under Assumption 1, a positive aggregate transitory demand shock θ generates capital inflows in period 0 and a boom-and-bust in period 1 and 2. A positive aggregate permanent shock ψ generates no capital flows and a long-lived boom in period 1 and 2.*

A temporary demand shock θ generates a demand boom in period 0 which makes households increase their borrowing. This same temporary demand boom makes firms mistakenly interpret it as a permanent boom, making them over-optimistic, which triggers a boom-and-bust dynamics in the non-tradable sector. A permanent increase in demand does not generate a boom-and-bust dynamics since firms are confirmed in their beliefs. At the same time, households do not borrow as the shock is permanent.

A.4 More dynamics

A caveat to our analysis is its static nature. We consider a dynamic extension of the benchmark model in order to study how the boom-and-bust pattern generalizes to a more standard dynamic framework. In particular, how long do booms and busts last? Besides, is the dynamic oscillatory, one boom generating a bust, then the bust generating a boom, etc.?

In order to map the model to standard DSGE models, we introduce sticky information à la Mankiw and Reis (2002). We also add noise to the observed mark-up for information not to be trivially revealed after a few periods. In this context, excessive optimism does not generate oscillations. It generates a temporary boom followed by a prolonged recession: optimism is reversed quickly but pessimism is long-lasting. More information frictions (either more sticky information or more noisy mark-ups) make the bust milder but more protracted.

We consider the full-fledged version of the model with the baseline calibration. There is an infinite number of periods, starting from $t = 1$. The household has the following lifetime utility:

$$\sum_{t=1}^{\infty} \beta^t U_t \tag{37}$$

which is a generalization of (1), where U_t follows

$$U_t = \Phi_t \frac{Q_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\eta}}{1+\eta}$$

$\phi_t = \log(\Phi_t)$ is a preference shifter that stays constant with probability $1 - p$:

$$\phi_t = \phi_{t-1}$$

and follows a random walk with probability p :

$$\phi_t = \phi_{t-1} + \psi_t$$

In that case, firms receive the signal $\psi_{int} = \psi_t + \theta_t + \Lambda_{nt} + \lambda_{int}$, that they can use when they update their information.

Each period, a firm has a probability δ to update its information, with $0 < \delta < 1$, in the spirit of sticky information models. However, unlike typical sticky information models, firms do not observe the state of fundamentals directly, but observe the set of available signals. This implies that at period t , a fraction δ of firms observes current and past values of ψ_{int} , the past sequence of mark-ups $\{p_{ins-1} - w_{ns-1}\}_{s \geq 0}$ and the current and past nominal wages $\{w_{ns}\}_{s \geq 0}$.

Note that if we allowed firms to observe $\{p_{ins-1} - w_{ns-1}\}_{s=1, \dots, t}$ exactly, then they could back out ϕ_t with only few observations. We therefore assume that the $p_{ins-1} - w_{ns-1}$ are observed with an idiosyncratic noise ω_{int} , with mean zero and standard error σ_ω . Firms can then never perfectly infer ϕ_t , but their expectations can get arbitrarily close to the realization of ϕ_t as information builds up over time.³

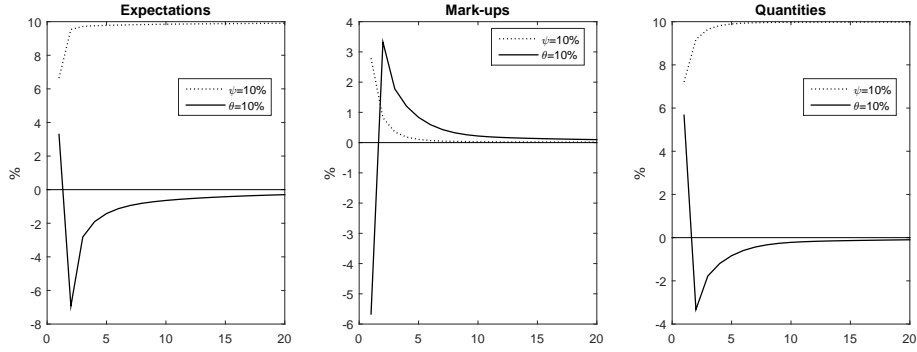


Figure 4: Impulse responses - Dynamic extension

We represent the effects of a 10% shock on θ ($\theta = 0.1$). Expectations are the expectations conditional on signals contemporaneously available. Variables are in deviations from the steady state.

We simulate this model using our baseline calibration. We additionally set $\delta = 0.5$, to match an average of two reviews per year (Blinder, 1998), and $\sigma_\omega = 0.001$ as a baseline.⁴ We represent the impulse response functions to a demand shock ψ_t and to a noise shock θ_t . Figure 4 shows the impulse response to a

³Note that the limit case where $\delta = 1$ and $\sigma_\omega = 0$, and a ψ shock occurs in period 1, boils down to our baseline 2-period model with additional periods. From $t = 4$ onwards, firms can perfectly deduce the value of ψ .

⁴The simulation procedure is described in the Online Appendix.

positive signal $\psi_{int} = 10\%$ when the underlying shock is a fundamental shock ($\psi_t = 10\%$) and when it is an optimism shock ($\theta_t = 10\%$). Consider first the effect of the fundamental shock. The dynamics features slow learning: expectations and quantities converge gradually to the actual value of ψ_t . More specifically, here quantities rise by 8% initially then slowly converge to 10%. Mark-ups initially increase because of the demand shock. They decrease gradually afterwards as supply is building up slowly. Consider now the noise shock. Initially, output increases by 6% as a result of optimism. But because of the excessive supply, the mark-up drops. Because this is a signal of low demand, expectations are reversed in the next period, which provokes a bust in quantities: output drops by 4% relative to its initial level, which constitutes a cumulated decline of 10%. The mark-up bounces back as well, which constitutes a positive signal to firms. This helps expectations and hence quantities revert back progressively to zero in the following periods. Note that the second reversal in information, contrary to the first one, is not strong enough to generate oscillations.

Figure 5 represents the behavior of output and expectations for different values of δ and σ_ω . Note that booms and busts do not occur when σ_ω is large enough (here, $\sigma_\omega = 0.01$). In that case, the mark-up signal is not reliable enough to generate a reversal in expectations, and the economy experiences a short-lived boom. For parameter values that generate a boom-and-bust, the peak-to-through drop in output is inversely proportional to the length of the bust. When σ_ω is lower, mark-ups are observed with a higher accuracy, which generates a stronger bust. In subsequent periods however, as the additional mark-up observations bring more accurate information, expectations quickly go back to zero, and output recovers faster. Similarly, when δ is larger, more firms observe the negative mark-ups, which generates a stronger bust as well, while in subsequent periods more firms revise their expectations upwards, which generates a faster recovery. Note that, with a higher δ , the boom is also larger, which, by generating a more negative mark-up signal, reinforces the bust. In general, more information frictions (lower δ , larger σ_ω) reduce the magnitude of the boom-and-bust but increase the persistence of the bust.

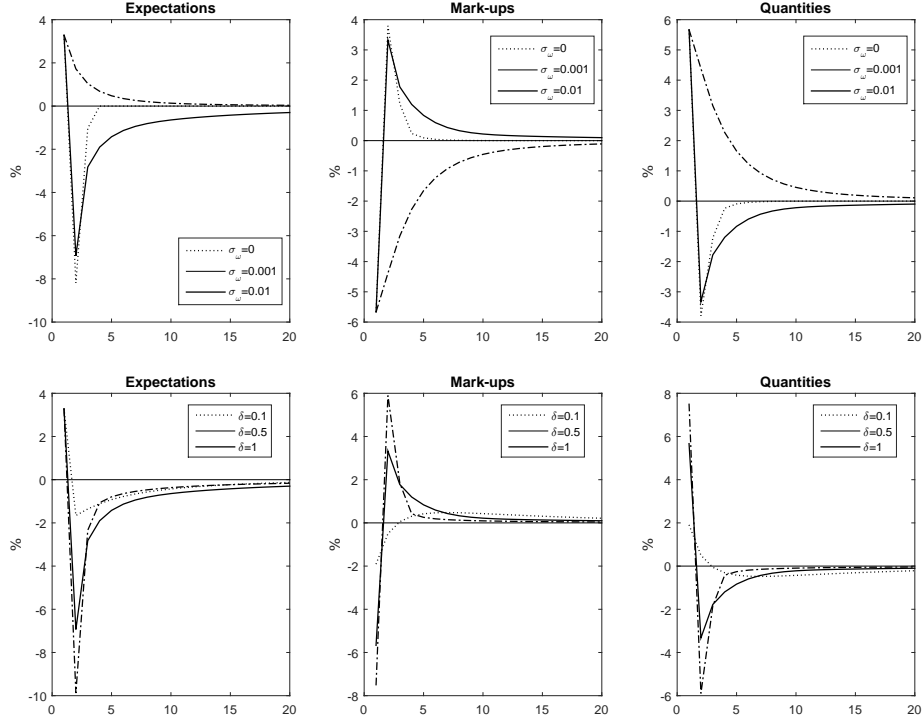


Figure 5: Impulse responses - Dynamic extension

We represent the effects of a 10% shock on θ ($\theta = 0.1$). Variables are in deviations from the steady state.

B Model with constant terms

Here we derive the models' equations with constant terms. Because shocks are log-normal, we have $q_{nt} = \int_0^1 q_{int} di + (1 - \rho)V_{it}(q_{int})/2$ and $q_t = \sum_{n=1}^N q_{nt}/N + (1 - \epsilon)V_{nt}(q_{nt})/2$, where $V_{nt}(\cdot)$ is the variance across sectors in period t and $V_{it}(\cdot)$ is the variance across firms within a sector in period t .

Equations (8), (9) and (11) are unchanged. However, now (10) admits a constant term:

$$w_{nt} - p_t = \sigma q_t + \chi(q_{nt} - q_t) - \psi - \zeta_t \quad (38)$$

where ζ_t is a time-dependent constant that is common knowledge to the firms, with $\zeta_t = (\eta - \chi)(1 - \epsilon)V_{nt}(q_{nt})/2 + \eta(1 - \rho)V_{it}(q_{int})/2$. Similarly, (12) becomes:

$$E_{int}(p_{int} - w_{nt} + \xi_t) = 0 \quad (39)$$

with $\xi_t = V_{int}(p_{int})$ and $V_{int}(\cdot) = V_{int}(\cdot | \Omega_{int})$ is the variance conditional on the

information of firm i in sector n and on period t .

B.1 The reduced-form model

As a result, the quantity-setting equation (13) is modified as follows:

$$\hat{q}_{int} = (1 + \kappa_b + \kappa_w)E_{int}(\psi) - \kappa_b E_{int}(\hat{q}_t) - \kappa_w E_{int}(\hat{q}_{nt}) \quad (40)$$

where we define $\hat{q}_{int} = \sigma q_{int} - \zeta_t - \xi_t + \phi_t$, $\hat{q}_{nt} = \int_0^1 \hat{q}_{int} di$ and $\hat{q}_t = \sum_{n=1}^N \hat{q}_{nt}/N$, with $\phi_t = \frac{1}{1+\kappa_b+\kappa_w} [(\kappa_b + \kappa_w)(1 - \rho)V_{it}(q_{int})/2 + \kappa_w(1 - \epsilon)V_{nt}(q_{nt})/2]$.

The endogenous signals (14) and (15) become:

$$w_{nt} = \left(1 - \frac{1}{\sigma}\right) \hat{q}_t + \frac{\chi}{\sigma} (\hat{q}_{nt} - \hat{q}_t) - \psi + m_t - \zeta_t + \frac{\gamma - 1}{\sigma} (\zeta_t + \xi_t - \phi_t) \quad (41)$$

$$p_{in1} - w_{n1} = \frac{1}{1 + \kappa_b + \kappa_w} \{(1 + \kappa_b + \kappa_w)\psi - \kappa_b \hat{q}_1 - \kappa_w \hat{q}_{n1} - q_{in1}\} + \zeta_t \quad (42)$$

The quantity-setting problem of the firms is exactly the same as in the paper, where we neglect the constant terms. Equation (40) is the same as (13), and the signals as expressed in (41) and (42) are equivalent to the signals as expressed in (14) and (15), as they are the same up to constant terms (ζ_t , ξ_t and ϕ_t) that are common knowledge to the firms.

B.2 The solution

After solving for \hat{q}_{int} , \hat{q}_{nt} and \hat{q}_t , we can recover the constant terms

$$\begin{aligned} \zeta_t &= \frac{(\eta - \chi)(1 - \epsilon)}{2\sigma^2} V_{nt}(\hat{q}_{nt}) + \frac{\eta(1 - \rho)}{2\sigma^2} V_{it}(\hat{q}_{int}) \\ \xi_t &= V_{int}(m_t) + \frac{(1 - \epsilon)^2}{\sigma^2} V_{int}(\hat{q}_t) + \frac{(\epsilon - \rho)^2}{\sigma^2} V_{int}(\hat{q}_{nt}) \\ \phi_t &= \frac{1}{1 + \kappa_b + \kappa_w} \left[\frac{(\kappa_b + \kappa_w)(1 - \rho)}{2\sigma^2} V_{it}(\hat{q}_{int}) + \frac{\kappa_w(1 - \epsilon)}{2\sigma^2} V_{nt}(\hat{q}_{nt}) \right] \end{aligned} \quad (43)$$

We can then obtain

$$\begin{aligned} q_{int} &= \frac{1}{\sigma} (\hat{q}_{int} + \zeta_t + \xi_t - \phi_t) \\ q_{nt} &= \frac{1}{\sigma} (\hat{q}_{nt} + \zeta_t + \xi_t - \phi_t) + \frac{(1 - \rho)}{2\sigma^2} V_{it}(\hat{q}_{int}) \\ q_t &= \frac{1}{\sigma} (\hat{q}_t + \zeta_t + \xi_t - \phi_t) + \frac{(1 - \epsilon)}{2\sigma^2} V_{nt}(\hat{q}_{nt}) + \frac{(1 - \rho)}{2\sigma^2} V_{it}(\hat{q}_{int}) \end{aligned} \quad (44)$$

C Main proofs

C.1 Preliminaries

Denote by ξ_i a gaussian vector of shocks of size N , where the n first elements are aggregate shocks and the $N - n$ last elements are idiosyncratic shocks, and S_i a vector of signals of size K such that there exists a (N, K) matrix H such that:

$$S_i = H'\xi_i \quad (45)$$

We denote by \tilde{H} the (N, K) matrix such that all the n first lines are equal to the n first lines of H and the $N - n$ last lines are equal to zero. Let P be a (N, K) matrix such that:

$$E(\xi_i|S_i) = PS_i \quad (46)$$

The following lemma will be useful in proving several propositions:

Lemma C.1. *Consider the following equation:*

$$\hat{q}_i = [(1 + \kappa_a)X'E(\xi_i|S_i) - \kappa_aE(\hat{q}|S_i)] \quad (47)$$

where X is a vector of size N . Then, if $I + \kappa_a\tilde{H}'P$ is invertible, we have:

$$\hat{q}_i = AS_i$$

where A is a size- K row vector such that:

$$A = (1 + \kappa_a)X'P[I + \kappa_a\tilde{H}'P]^{-1} \quad (48)$$

Proof of Lemma C.1. We use the method of undetermined coefficients to solve for A . We first form the educated guess that there exist a size- K row vector A such that

$$\hat{q}_i = AS_i \quad (49)$$

then, using Equation (45), we obtain

$$\hat{q}_i = AH'\xi_i$$

Hence, aggregating across firms, taking expectations and using Equation (46):

$$E(\hat{q}|S_i) = A\tilde{H}'E(\xi_i|S_i) = A\tilde{H}'PS_i$$

Replacing in Equation (47):

$$\hat{q}_i = [(1 + \kappa_a)X'PS_i - (\tilde{\sigma} - 1)A\tilde{H}'PS_i] = [(1 + \kappa_a)X'P - \kappa_aA\tilde{H}'P]S_i$$

Using the guess, we can write:

$$A = (1 + \kappa_a)X'P - \kappa_aA\tilde{H}'P$$

If $I + \kappa_a\tilde{H}'P$ is invertible, we can solve for A and obtain (48). □

C.2 Derivation of Equations (21) and (22)

According to Equation (13), \hat{q}_{i1} follows (47) with $S_i = \psi_i$, $\xi_i = (\psi \ \theta \ \lambda_i)'$ and $X = (1 \ 0 \ 0)'$. Besides, S_i follows (45) with $H = (1 \ 1 \ 1)'$ and $\tilde{H} = (1 \ 1 \ 0)'$; and $E(\xi_i|S_i)$ follows (46) with $P = (k_\psi \ \bar{k}_\psi \ 1 - k_\psi - \bar{k}_\psi)'$ with $k_\psi = \sigma_\psi^2/(\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2)$ and $\bar{k}_\psi = \sigma_\theta^2/(\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2)$. Therefore, applying Lemma C.1, we obtain:

$$K_\psi = A = \frac{(1 + \kappa_a)k_\psi}{1 + \kappa_a(k_\psi + \bar{k}_\psi)} = k_\psi \left(1 + \frac{\kappa_a(1 - k_\psi - \bar{k}_\psi)}{1 + \kappa_a(k_\psi + \bar{k}_\psi)} \right)$$

We have: $K_\psi = (1 + \kappa_a)k_\psi/[(1 + \kappa_a)(k_\psi + \bar{k}_\psi) + (1 - k_\psi - \bar{k}_\psi)]$, which implies $0 < K_\psi < 1$ as $k_\psi + \bar{k}_\psi < 1$. Besides, we have $k_\psi + \bar{k}_\psi < 1$ and under Assumption 1 $\kappa_a > 0$, so $K_\psi > k_\psi$.

C.3 Derivation of Equation (23)

The standard signal extraction formula gives us that $E_{i2}(\psi) = f_x x_i + f_s s$ with

$$f_x = \frac{(\omega_\lambda \sigma_\lambda)^{-2}}{(\sigma_\psi)^{-2} + (\omega_\theta \sigma_\theta)^{-2} + (\omega_\lambda \sigma_\lambda)^{-2}} = \frac{(1 + \omega_\theta)^2 \sigma_\psi^2 \sigma_\theta^2}{(1 + \omega_\theta)^2 \sigma_\psi^2 \sigma_\theta^2 + \sigma_\psi^2 \sigma_\lambda^2 + \omega_\theta^2 \sigma_\theta^2 \sigma_\lambda^2}$$

$$f_s = \frac{(\omega_\theta \sigma_\theta)^{-2}}{(\sigma_\psi)^{-2} + (\omega_\theta \sigma_\theta)^{-2} + (\omega_\lambda \sigma_\lambda)^{-2}} = \frac{\sigma_\psi^2 \sigma_\lambda^2}{(1 + \omega_\theta)^2 \sigma_\psi^2 \sigma_\theta^2 + \sigma_\psi^2 \sigma_\lambda^2 + \omega_\theta^2 \sigma_\theta^2 \sigma_\lambda^2}$$

where we used $\omega_\lambda = \omega_\theta/(1 + \omega_\theta)$. Obviously, $0 < f_x < 1$, $0 < f_s < 1$ and $f_x + f_s < 1$. Besides, we can show that $f_x + f_s > k_\psi$. Indeed, using the definitions of f_x , f_s and k_ψ , we can show that this is equivalent to: $[(\sigma_\theta^2 + \sigma_\lambda^2)(1 + \omega_\theta) - \sigma_\lambda^2 \omega_\theta]^2 > 0$, which is always the case. Finally, we can show that f_s is decreasing in K_ψ as $\omega_\theta < 1$. Since ω_θ is increasing in κ_a , then f_s is decreasing in $\tilde{\sigma}$.

C.4 Derivation of Equations (24) and (25)

According to Equation (13), \hat{q}_{i2} follows (47) with $S_i = \begin{pmatrix} s & x_i \end{pmatrix}'$,

$$\xi_i = \begin{pmatrix} \psi \\ -\omega_\theta \theta \\ \omega_\lambda \lambda_i \end{pmatrix}$$

and $X = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}'$. Besides, S_i follows (45) with

$$H = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\tilde{H} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and $E(\xi_i|S_i)$ follows (46) with

$$P = \begin{pmatrix} f_s & f_x \\ 1 - f_s & -f_x \\ -f_s & 1 - f_x \end{pmatrix}$$

with f_s and f_x defined as in the derivation of Equation (23). Therefore, applying Lemma C.1, we obtain:

$$\begin{pmatrix} F_s \\ F_x \end{pmatrix} = A' = \begin{pmatrix} \frac{f_s}{1+\kappa_a f_x} \\ \frac{(1+\kappa_a)f_x}{1+\kappa_a f_x} \end{pmatrix} = \begin{pmatrix} f_s \left(1 - \frac{\kappa_a f_x}{1+\kappa_a f_x}\right) \\ f_x \left(1 + \frac{\kappa_a(1-f_x)}{1+\kappa_a f_x}\right) \end{pmatrix}$$

We have $0 < f_x < 1$ and according to Assumption 1, we have $\kappa_a > 0$, so $F_x > f_x$ and $F_s < f_s$. Besides, one can show that $F_x + F_s > K_\psi$. Indeed, using the definitions of F_x , F_s , K_ψ and ω_λ , we can show that this is equivalent to: $[(1 + \kappa_a)\sigma_\theta^2 + \sigma_\lambda^2](1 + \omega_\theta) - \sigma_\lambda^2 \omega_\theta > 0$, which is always the case. Finally, we have $F_x + F_s = [f_s + (1 + \kappa_a)f_x]/[1 - f_x + (1 + \kappa_a)f_x]$. Since $f_x + f_s < 1$, then $F_x + F_s < 1$.

C.5 Proof of Corollary 1

Using the definition of f_s given in the derivation of Equation (23), it is straightforward to see that f_s goes to 1 and f_x goes to zero as σ_θ goes to zero. As a result, following the definition of F_s given in the derivation of Equations (24) and (25), we show that F_s goes to f_s as σ_θ goes to zero.

Hence, as σ_θ goes to zero, $-F_s\omega_\theta$ goes to $-f_s\omega_\theta$ which goes to $-\kappa_a\sigma_\psi^2/(\sigma_\psi^2 + \sigma_\lambda^2)$.

C.6 Proof of Proposition 3

When there is both idiosyncratic and sectoral noise, the problem is more complicated because the exogenous signal ψ_{in} and the endogenous signal $p_{in1} - w_{n1}$ cannot be combined into independent signals, as we have

$$\psi_{in} = \psi + \theta + \Lambda_n + \lambda_{in}$$

$$p_{in1} - w_{n1} + \frac{1}{1 + \kappa_a}q_{in1} = \left(1 - \frac{\kappa_a}{1 + \kappa_a}K_\psi\right)\psi - \frac{\kappa_a}{1 + \kappa_a}K_\psi\theta - \frac{\kappa_w}{1 + \kappa_a}K_\psi\Lambda_n$$

where we have used the expression for the endogenous signal (15) and $q_{in1} = K_\psi\psi_{in}$, with $0 < K_\psi < 1$.

The updating of expectations goes as follows:

$$E_{in2}(\psi) = E_{in1}(\psi) + \frac{Corr[s - E_{in1}(s), \psi - E_{in1}(\psi)]}{Var[s - E_{in1}(s)]}[s - E_{in1}(s)]$$

where $s = \psi - \frac{\kappa_a}{1 + \kappa_a - \kappa_a K_\psi}K_\psi\theta - \frac{\kappa_w}{1 + \kappa_a - \kappa_a K_\psi}K_\psi\Lambda_n$ is the normalized endogenous signal. We denote $s = \psi - \alpha\theta - \beta\Lambda_n$ for simplicity. We show that the coefficient of θ in this expression is negative if and only if Condition 3 is satisfied (see the full proof in Section D).

C.7 Proof of Proposition A.1

The proof proceeds in two steps.

Capital flows First, we show that a positive temporary demand shock generates an increase in the consumption of tradable goods in period 0 relative to period 1 and 2. Households have then to borrow in period 0 and reimburse their debt in period 1 and 2. On the opposite, with a positive permanent demand shock, households consume the same amount during the three periods and do not borrow. See Section D for details.

Period-1 and period-2 production For periods 1 and 2 Equations (22) and (25) of the simple model hold, so period 1 and 2 feature the same dynamics as the simple model.

D Additional proofs

D.1 Proof of Proposition 3 (full derivations)

When there is both idiosyncratic and sectoral noise, the problem is more complicated because the exogenous signal ψ_{in} and the endogenous signal $p_{in1} - w_{n1}$ cannot be combined into independent signals, as we have

$$\begin{aligned} \psi_{in} &= \psi + \theta + \Lambda_n + \lambda_{in} \\ p_{in1} - w_{n1} + \frac{1}{1 + \kappa_a} q_{in1} &= \left(1 - \frac{\kappa_a}{1 + \kappa_a} K_\psi\right) \psi - \frac{\kappa_a}{1 + \kappa_a} K_\psi \theta - \frac{\kappa_w}{1 + \kappa_a} K_\psi \Lambda_n \end{aligned}$$

where we have used the expression for the endogenous signal (15) and $q_{in1} = K_\psi \psi_{in}$, with $0 < K_\psi < 1$.

The updating of expectations goes as follows:

$$E_{in2}(\psi) = E_{in1}(\psi) + \frac{\text{Corr}[s - E_{in1}(s), \psi - E_{in1}(\psi)]}{\text{Var}[s - E_{in1}(s)]} [s - E_{in1}(s)]$$

where $s = \psi - \frac{\kappa_a}{1 + \kappa_a - \kappa_a K_\psi} K_\psi \theta - \frac{\kappa_w}{1 + \kappa_a - \kappa_a K_\psi} K_\psi \Lambda_n$ is the normalized endogenous signal. We denote $s = \psi - \alpha\theta - \beta\Lambda_n$ for simplicity. We show in what follows that the coefficient of θ in this expression is negative if and only if Condition 3 is satisfied.

Using $E_{in1}(\psi) = k_\psi \psi_i$ and $E_{in1}(s) = (k_\psi - \alpha k_\theta - \beta k_\Lambda) \psi_i$, with $k_\psi = \sigma_\psi^2 / [\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\Lambda^2 + \sigma_\lambda^2]$, $k_\theta = \sigma_\theta^2 / [\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\Lambda^2 + \sigma_\lambda^2]$ and $k_\Lambda = \sigma_\Lambda^2 / [\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\Lambda^2 + \sigma_\lambda^2]$, we get

$$\begin{aligned} E_{in2}(\psi) &= k_\psi \psi_i + \frac{\text{Corr}[s - E_{in1}(s), \psi - E_{in1}(\psi)]}{\text{Var}[s - E_{in1}(s)]} [s - (k_\psi - \alpha k_\theta - \beta k_\Lambda) \psi_i] \\ &= k_\psi (\psi + \theta + \Lambda_n + \lambda_{in}) + \\ &\quad \frac{\text{Corr}[s - E_{in1}(s), \psi - E_{in1}(\psi)]}{\text{Var}[s - E_{in1}(s)]} \left([1 - k_\psi + \alpha k_\theta + \beta k_\Lambda] \psi + \right. \\ &\quad \left. [-k_\psi - \alpha(1 - k_\theta) + \beta k_\Lambda] \theta + [-k_\psi + \alpha k_\theta - \beta(1 - k_\Lambda)] \Lambda_n + \right. \\ &\quad \left. [-k_\psi + \alpha k_\theta + \beta k_\Lambda] \lambda_{in} \right) \end{aligned}$$

The coefficient of θ in $E_{in2}(\psi)$ is thus

$$k_\psi - \frac{\text{Corr}[s - E_{in1}(s), \psi - E_{in1}(\psi)]}{\text{Var}[s - E_{in1}(s)]} [-k_\psi - \alpha(1 - k_\theta) + \beta k_\Lambda]$$

This term is negative if and only if

$$k_\psi \text{Var}[s - E_{in1}(s)] < [k_\psi + \alpha(1 - k_\theta) - \beta k_\Lambda] \text{Corr}[s - E_{in1}(s), \psi - E_{in1}(\psi)] \quad (50)$$

Then we use

$$\begin{aligned} \text{Corr}[s - E_{in1}(s), \psi - E_{in1}(\psi)] &= (1 - k_\psi)[1 - k_\psi + \alpha k_\theta + \beta k_\Lambda] \sigma_\psi^2 \\ &\quad - k_\psi[-k_\psi - \alpha(1 - k_\theta) + \beta k_\Lambda] \sigma_\theta^2 \\ &\quad - k_\psi[-k_\psi + \alpha k_\theta - \beta(1 - k_\Lambda)] \sigma_\Lambda^2 \\ &\quad - k_\psi[-k_\psi + \alpha k_\theta + \beta k_\Lambda] \sigma_\lambda^2 \end{aligned}$$

$$\begin{aligned} V[s - E_{in1}(s)] &= [1 - k_\psi + \alpha k_\theta + \beta k_\Lambda]^2 \sigma_\psi^2 \\ &\quad + [-k_\psi - \alpha(1 - k_\theta) + \beta k_\Lambda]^2 \sigma_\theta^2 \\ &\quad + [-k_\psi + \alpha k_\theta - \beta(1 - k_\Lambda)]^2 \sigma_\Lambda^2 \\ &\quad + [-k_\psi + \alpha k_\theta + \beta k_\Lambda]^2 \sigma_\lambda^2 \end{aligned}$$

After replacing in (50), and using the definition of k_ψ , k_θ and k_Λ , we get

$$\begin{aligned} &[1 - k_\psi + \alpha k_\theta + \beta k_\Lambda]^2 k_\psi \\ &+ [-k_\psi - \alpha(1 - k_\theta) + \beta k_\Lambda]^2 k_\theta \\ &+ [-k_\psi + \alpha k_\theta - \beta(1 - k_\Lambda)]^2 k_\Lambda \\ &+ [-k_\psi + \alpha k_\theta + \beta k_\Lambda]^2 (1 - k_\psi - k_\theta - k_\Lambda) \\ &< [k_\psi + \alpha(1 - k_\theta) - \beta k_\Lambda] \cdot \\ &\quad [(1 - k_\psi)[1 - k_\psi + \alpha k_\theta + \beta k_\Lambda] \\ &\quad - [-k_\psi - \alpha(1 - k_\theta) + \beta k_\Lambda] k_\theta \\ &\quad - [-k_\psi + \alpha k_\theta - \beta(1 - k_\Lambda)] k_\Lambda \\ &\quad - [-k_\psi + \alpha k_\theta + \beta k_\Lambda] k_\lambda] \\ &\Leftrightarrow [1 - X]^2 k_\psi + [-\alpha - X]^2 k_\theta + [-\beta - X]^2 k_\Lambda + [-X]^2 (1 - k_\psi - k_\theta - k_\Lambda) \\ &< [k_\psi + \alpha(1 - k_\theta) - \beta k_\Lambda] \cdot [(1 - k_\psi)[1 - X] - [-\alpha - X] k_\theta - [-\beta - X] k_\Lambda - [-X](1 - k_\psi - k_\theta - k_\Lambda)] \end{aligned}$$

with $X = k_\psi - \alpha k_\theta - \beta k_\Lambda$.

$$\begin{aligned} &\Leftrightarrow [1 - X][(1 - k_\psi)(\alpha + X) - k_\psi(1 - X)] + (\alpha + X)[k_\theta(\alpha + X) - k_\theta(\alpha + X)] \\ &+ (\beta + X)[k_\Lambda(\alpha + X) - k_\Lambda(\beta + X)] + X(1 - k_\psi - k_\theta - k_\Lambda)[\alpha + X - X] > 0 \end{aligned}$$

$$\begin{aligned} &\Leftrightarrow [1 - X][\alpha + X - k_\psi(1 + \alpha)] \\ &+ (\alpha - \beta)(\beta + X)k_\Lambda + \alpha X(1 - k_\psi - k_\theta - k_\Lambda) > 0 \end{aligned}$$

Replacing X , we get:

$$\begin{aligned} &[1 - k_\psi + \alpha k_\theta + \beta k_\Lambda][\alpha(1 - k_\psi - k_\theta - k_\Lambda) + (\alpha - \beta)k_\Lambda] \\ &+ (\alpha - \beta)[k_\psi - \alpha k_\theta + \beta(1 - k_\Lambda)]k_\Lambda + \alpha(k_\psi - \alpha k_\theta - \beta k_\Lambda)(1 - k_\psi - k_\theta - k_\Lambda) > 0 \end{aligned}$$

$$\Leftrightarrow \alpha(1 - k_\psi - k_\theta - k_\Lambda) + (\alpha - \beta)(\beta + 1)k_\Lambda > 0$$

Using the definition of k_ψ , k_θ and k_Λ :

$$\alpha\sigma_\lambda^2 + (\alpha - \beta)(\beta + 1)\sigma_\Lambda^2 > 0$$

Replacing α and β by their values, we obtain Condition 3, hence the result.

D.2 Additional public signal

According to Equation (13), \hat{q}_{i1} follows (47) with $S_i = (\psi_e \ \psi_i)'$,

$$\xi_i = \begin{pmatrix} \psi \\ \theta \\ e \\ \lambda_i \end{pmatrix}$$

and $X = (1 \ 0 \ 0 \ 0)'$. Besides, S_i follows (45) with

$$H = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\tilde{H} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and $E(\xi_i|S_i)$ follows (46) with

$$P = \begin{pmatrix} k_e^* & k_\psi^* \\ -\tau k_e^* & \tau(1 - k_\psi^*) \\ 1 - k_e^* & -k_\psi^* \\ -(1 - \tau)k_e^* & (1 - \tau)(1 - k_\psi^*) \end{pmatrix}$$

with k_e^* and k_ψ^* defined as follows:

$$k_\psi^* = \frac{(\sigma_\theta^2 + \sigma_\lambda^2)^{-1}}{(\sigma_\psi)^{-2} + (\sigma_\theta^2 + \sigma_\lambda^2)^{-1} + (\sigma_e)^{-2}} = \frac{\sigma_\psi^2}{\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2 + \sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/\sigma_e^2}$$

$$k_e^* = \frac{(\sigma_e)^{-2}}{(\sigma_\psi)^{-2} + (\sigma_\theta^2 + \sigma_\lambda^2)^{-1} + (\sigma_e)^{-2}} = \frac{\sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/\sigma_e^2}{\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2 + \sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/\sigma_e^2}$$

and $\tau = \sigma_\theta^2/(\sigma_\theta^2 + \sigma_\lambda^2)$. Therefore, applying Lemma C.1, we obtain:

$$A' = \begin{pmatrix} \frac{[1+\tau\kappa_a]k_e^*}{1+\kappa_a[k_\psi^* + \tau(1-k_\psi^*)]} \\ \frac{(1+\kappa_a)k_\psi^*}{1+\kappa_a[k_\psi^* + \tau(1-k_\psi^*)]} \end{pmatrix}$$

We can write

$$A' = \begin{pmatrix} (1-\alpha)K_\psi \\ \alpha \end{pmatrix}$$

with K_ψ defined as before and $\alpha = \frac{\delta}{\sigma_e^2 + \delta}$ where $\delta = [1 + \tau\kappa_a]\sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/[(1 + \kappa_a)(\sigma_\psi^2 + \sigma_\theta^2) + \sigma_\lambda^2] > 0$. This yields Equation (33).

D.3 Additional private signal

According to Equation (13), \hat{q}_{i1} follows (47) with $S_i = (\psi_{ui} \ \psi_i)'$,

$$\xi_i = \begin{pmatrix} \psi \\ \theta \\ u_i \\ \lambda_i \end{pmatrix}$$

and $X = (1 \ 0 \ 0 \ 0)'$. Besides, S_i follows (45) with

$$H = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\tilde{H} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

and $E(\xi_i|S_i)$ follows (46) with

$$P = \begin{pmatrix} k_u^* & k_\psi^* \\ -\tau k_u^* & \tau(1-k_\psi^*) \\ 1-k_u^* & -k_\psi^* \\ -(1-\tau)k_u^* & (1-\tau)(1-k_\psi^*) \end{pmatrix}$$

with k_w^* and k_ψ^* defined as follows:

$$k_\psi^* = \frac{(\sigma_\theta^2 + \sigma_\lambda^2)^{-1}}{(\sigma_\psi)^{-2} + (\sigma_\theta^2 + \sigma_\lambda^2)^{-1} + (\sigma_u)^{-2}} = \frac{\sigma_\psi^2}{\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2 + \sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/\sigma_u^2}$$

$$k_u^* = \frac{(\sigma_u)^{-2}}{(\sigma_\psi)^{-2} + (\sigma_\theta^2 + \sigma_\lambda^2)^{-1} + (\sigma_u)^{-2}} = \frac{\sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/\sigma_u^2}{\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2 + \sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/\sigma_u^2}$$

and $\tau = \sigma_\theta^2/(\sigma_\theta^2 + \sigma_\lambda^2)$. Therefore, applying Lemma C.1, we obtain:

$$A' = \begin{pmatrix} \frac{(1+\kappa_a)[1+\tau\kappa_a]k_u^*}{1+\kappa_a[k_\psi^* + \tau(1-k_\psi^*) + (1+\tau\kappa_a)k_u^*]} \\ \frac{(1+\kappa_a)k_\psi^*}{1+\kappa_a[k_\psi^* + \tau(1-k_\psi^*) + (1+\tau\kappa_a)k_u^*]} \end{pmatrix}$$

We can write

$$A' = \begin{pmatrix} (1-\alpha)K_\psi \\ \alpha \end{pmatrix}$$

with $\alpha = \frac{\delta}{\sigma_u^2 + \delta}$ where $\delta = (1+\kappa_a)[1+\tau\kappa_a]\sigma_\psi^2(\sigma_\theta^2 + \sigma_\lambda^2)/[(1+\kappa_a)(\sigma_\psi^2 + \sigma_\theta^2) + \sigma_\lambda^2] > 0$. This yields Equation (33).

D.4 Proof of Proposition A.1 (full derivations)

Capital flows The household maximizes $U = U_0 + \beta U_1 + \beta^2 U_2$, where U_0 , U_1 and U_2 are defined as in Section 4, subject to the following intertemporal budget constraint, expressed in terms of traded goods, with the terminal condition $D_2 = 0$ and the initial condition $D_{-1} = 0$:

$$\begin{aligned} \frac{P_0}{P_0^*}Q_0 + \frac{P_1}{rP_1^*}Q_1 + \frac{P_2}{r^2P_2^*}Q_2 + X_0 + \frac{X_1}{r} + \frac{X_2}{r^2} &= \frac{\Pi_0 + W_0L_0}{P_0^*} + \frac{\Pi_1 + W_1L_1}{rP_1^*} + \frac{\Pi_2 + W_2L_2}{r^2P_2^*} \\ &+ \left(1 + \frac{1}{r} + \frac{1}{r^2}\right)\bar{X} \end{aligned}$$

As $r = 1/\beta$, the Euler equations for X yield:

$$\begin{aligned} (1-\mu)e^\theta X_0^{-1} &= (1-\mu)X_1^{-1} \\ &= (1-\mu)X_2^{-1} \end{aligned}$$

After rearranging, we obtain:

$$\frac{1}{e^\theta}X_0 = X_1 = X_2 \tag{51}$$

On the other hand, using the definition of profits in the non-traded sector $\Pi_t = P_t Q_t - W_t L_t$ and $r = 1/\beta$, we obtain the consolidated budget constraint for traded goods:

$$X_0 + \beta X_1 + \beta^2 X_2 = (1 + \beta + \beta^2) \bar{X}$$

Replacing X_1 and X_2 using (51), we get the consumptions of traded goods:

$$X_0 = \frac{(1 + \beta + \beta^2) \bar{X} e^\theta}{\beta + \beta^2 + e^\theta}, \quad X_1 = X_2 = \frac{(1 + \beta + \beta^2) \bar{X}}{\beta + \beta^2 + e^\theta}$$

It is then straightforward to derive the evolution of debt:

$$D_0 = X_0 - \bar{X} = \frac{(\beta + \beta^2)(e^\theta - 1)}{\beta + \beta^2 + e^\theta} \bar{X}, \quad D_1 = r D_0 + X_1 - \bar{X} = \beta \frac{e^\theta - 1}{\beta + \beta^2 + e^\theta} \bar{X}$$

A positive aggregate transitory shock $\theta > 0$ therefore generates a capital inflow in period 0 as $D_0 > 0$. In period 1 there is a capital outflow and the debt level diminishes $0 < D_1 < D_0$. In period 2 the household reimburses the remaining debt ($D_2 = 0$).

Period-1 and period-2 production For periods 1 and 2 Equations (22) and (25) of the simple model hold, so period 1 and 2 feature the same dynamics as the simple model.

E Numerical simulations

We describe here the iterative procedure to simulate the T -period dynamic model of Section 4, when normalizing initial expectations of the demand shock to zero, and when a demand innovation ψ occurs in period 1. The 2-period model of Section 3 is then simulated as a special case. It corresponds to the case with $\delta = 1$, $\sigma_\epsilon = 0$ and $T = 2$.

To describe the simulation procedure for the dynamic model, we proceed in two steps. First, we generalize Lemma C.1 to the case with sectoral noise where quantity-setting follows (13), and where, as it is the case in sticky-information models, firms have heterogeneous information sets. Second, we describe the iterative procedure used to solve the model when including the nominal wage as a signal. Indeed, in that case, the equilibrium production depends on the information structure, but the information structure depends on the equilibrium production.

We simulate here the dynamic economy for T periods. Denote by

$$\xi_{in} = \left(\psi \quad \theta \quad m_1 \quad \dots \quad m_T \quad \Lambda_n \quad \lambda_{in} \quad \epsilon_{in1} \quad \dots \quad \epsilon_{inT} \right)'$$

our gaussian vector of shocks. It is of size $2T + 4$, where the $2 + T$ first elements are aggregate shocks and the next $(3 + T^{th})$ element is a sectoral shock and the last $T + 1$ elements are idiosyncratic shocks. We then define \tilde{I} is the matrix that selects only aggregate shocks while \bar{I} selects aggregate and sectoral shocks. Namely, \tilde{I} has 1 in the $2 + T$ first elements of the diagonal, and zeros elsewhere, while \bar{I} has 1 in the first $3 + T$ elements of the diagonal and zero elsewhere.

Denote by S_{int} the vector of signals available to firms that update their signal in t . We have

$$S_{int} = \left(\psi_{in} \quad w_{n1} \quad p_{in1} - w_{n1} + \epsilon_{in2} \quad w_{n2} \quad \dots \quad p_{int-1} - w_{nt-1} + \epsilon_{int} \quad w_{nt} \right)'$$

our vectors of signals for the first and subsequent periods. We can write S_{int} as

$$S_{int} = H_t' \xi_{in} \tag{52}$$

As before, we denote by Σ the matrix of variance-covariance of ξ_{in} .

Consider the expectations of a firm that update her information in t . We have

$$E_{int}(\xi_{in}) = P_t H_t' \xi_{in} \tag{53}$$

with $P_t = \Sigma H_t (H_t' \Sigma H_t)^{-1}$.

Define \hat{q}_{int}^{t-h} as the supply in t by firms who update their information in $t - h$. We have

$$\hat{q}_{int}^{t-h} = (1 + \kappa_b + \kappa_w) E_{int-h}(\psi) - \kappa_b E_{int-h}(\hat{q}_t) - \kappa_w E_{int-h}(\hat{q}_{nt}) \tag{54}$$

with $\hat{q}_t = \sum_{h=0}^{t-1} \delta(1-\delta)^h \hat{q}_t^{t-h}$ and $\hat{q}_{nt} = \sum_{h=0}^{t-1} \delta(1-\delta)^h \hat{q}_{nt}^{t-h}$, where $\hat{q}_{nt}^{t-h} = \int_0^1 \hat{q}_{int}^{t-h} di$ and $\hat{q}_t^{t-h} = \frac{1}{N} \sum \hat{q}_{nt}^{t-h}$.

Lemma E.1. *Consider the following equation:*

$$\hat{q}_{int}^{t-h} = [(1 + \kappa_a) X' E(\xi_{in} | S_{int-h}) - \kappa_b E(\hat{q}_t | S_{int-h}) - \kappa_w E(\hat{q}_{nt} | S_{int-h})] \tag{55}$$

where X is a vector of size N . Denote by Π_t the average expectations in t :

$$\Pi_t = \sum_{h=0}^{t-1} \delta(1-\delta)^h P_{t-h} H_{t-h}'$$

Then, if $I + \kappa_b \Pi_t \tilde{I} + \kappa_w \Pi_t \bar{I}$ is invertible, we have:

$$\hat{q}_{int}^{t-h} = A_t P_{t-h} H'_{t-h} \xi_{in}$$

where A_t is a size- N row vector such that:

$$A_t = (1 + \kappa_a) X' P [I + \kappa_b \Pi_t \tilde{I} + \kappa_w \Pi_t \bar{I}]^{-1} \quad (56)$$

Then $\hat{q}_t = A_t \Pi_t \tilde{I}$ and $\hat{q}_{nt} = A_t \Pi_t \bar{I}$.

Proof of Lemma E.1. We use the method of undetermined coefficients to solve for A . We first form the educated guess that there exists a size- N row vector A_t such that

$$\hat{q}_{int}^{t-h} = A_t E_{int-h}(\xi_{in}) \quad (57)$$

then, using Equation (52), we obtain

$$\hat{q}_{int}^{t-h} = A_t P_{t-h} H'_{t-h} \xi_{in}$$

Hence, aggregating across firms, we obtain

$$\begin{aligned} \hat{q}_t &= \sum_{h=0}^{t-1} \delta(1-\delta)^h \hat{q}_t^{t-h} \\ &= \sum_{h=0}^{t-1} \delta(1-\delta)^h A_t P_{t-h} H'_{t-h} \tilde{I} \xi_{in} \\ &= A_t \left(\sum_{h=0}^{t-1} \delta(1-\delta)^h A_t P_{t-h} H'_{t-h} \right) \tilde{I} \xi_{in} \\ &= A_t \Pi_t \tilde{I} \xi_{in} \end{aligned}$$

Similarly,

$$\hat{q}_{nt} = A_t \Pi_t \bar{I} \xi_{in}$$

Taking expectations and using Equation (53):

$$E(\hat{q}_t | S_{int-h}) = A_t \Pi_t \tilde{I} P_{t-h} H'_{t-h} \xi_{in}$$

$$E(\hat{q}_{nt} | S_{int-h}) = A_t \Pi_t \bar{I} P_{t-h} H'_{t-h} \xi_{in}$$

Replacing in Equation (55):

$$\hat{q}_{int}^{t-h} = [(1 + \kappa_a) X' - \kappa_b A_t \Pi_t \tilde{I} - \kappa_w A_t \Pi_t \bar{I}] P_{t-h} H'_{t-h} \xi_{in}$$

Using the guess, we can write:

$$A_t = (1 + \kappa_a) X' - \kappa_b A_t \Pi_t \tilde{I} - \kappa_w A_t \Pi_t \bar{I}$$

If $I + \kappa_b \Pi_t \tilde{I} + \kappa_w \Pi_t \bar{I}$ is invertible, we can solve for A_t and obtain (56). □

Finally, define \bar{W}_t as the $1 \times (4 + 2T)$ vector with -1 as the first element and 1 as the $2 + t^{th}$ element.

We design an iterative procedure to solve for the model at each period.

Period 1 Consider the portion δ of firms that update their information at date 1.

At each step k of the iteration, we have $S_{in1}(k) = H_1(k)' \xi_{in}$ with:

$$H_1(k) = \begin{pmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} W_1(k)$$

where $W_1(k)$ is such that $w_{n1}(k) = W_1(k)' \xi_{in}$.

Define

$$P_1(k) = \Sigma H_1(k) (H_1(k)' \Sigma H_1(k))^{-1}$$

so that, for a given signal structure $H_1(k)$, $E_{in}(\xi_{in} | S_{in1}(k)) = P_1(k) S_{in1}(k) = P_1(k) H_1(k)' \xi_{in}$.

Denote by $A_1(k)$ the solution given by lemma E.1 for $H = H_1(k)$ and X equal to a size- N vector with 1 as the first element and zero elsewhere. We have aggregate output $\hat{q}_1(k) = A_1(k) \Pi_1 \tilde{I} \xi_{in}$ and $\hat{q}_{1n}(k) = A_1(k) \Pi_1(k) \bar{I} \xi_{in}$ with $\Pi_1(k) = \delta P_1(k) H_1(k)'$.

We update our guess for the wage as follows, using Equation (10):

$$W_1(k+1) = \left(\frac{1}{\sigma} - 1 \right) A_1(k) \Pi_1(k) \tilde{I} + \frac{\chi}{\sigma} A_1(k) \Pi_1(k) (\bar{I} - \tilde{I}) + \bar{W}_1$$

We set our initial guess for the nominal wage as $W_1(0) = \bar{W}_1$, which corresponds to the value of w_{n1} when $\sigma = 1$ and $\chi = 0$, according to (10). We stop at $k = K_1$ when $A_1(K_1)$ has converged and denote $A_1 = A_1(K_1)$, $H_1 = H_t(K_1)$ and $\Pi_1 = \Pi_1(K_1)$.

We compute

$$M_1 = \frac{1}{1 + \kappa_b + \kappa_w} \left([1 + \kappa_b + \kappa_w] X' - \kappa_b A_1(k) \Pi_1 \tilde{I} - \kappa_w A_1(k) \Pi_1 \bar{I} - A_1 \right)$$

where we used Equation (15).

Period $t > 1$ Consider any period $t > 1$. We proceed as for period 1. At each step k of the iteration, we have $S_{int}(k) = H_t(k)' \xi_{in}$ with:

$$H_t(k) = \begin{pmatrix} H_{t-1} & M_{t-1} & W_t(k) \end{pmatrix}$$

where $w_{nt}(k) = W_t(k)' \xi_{in}$ and $p_{int-1} - w_{nt-1}(k) = M_{t-1}' \xi_{in}$.

We define

$$P_t(k) = \Sigma H_t(k) (H_t(k)' \Sigma H_t(k))^{-1}$$

Finally, define $A_t(k)$ as the solution given by lemma E.1 for H_1, \dots, H_{t-1} and $H_t = H_t(k)$. We obtain actual sectoral and aggregate output $\hat{q}_t(k) = A_t(k) \Pi_t(k) \tilde{I}$ and $\hat{q}_{nt}(k) = A_t(k) \Pi_t(k) \bar{I}$ using $\Pi_t = \delta P_t(k) H_t(k)' + (1 - \delta) \Pi_{t-1}$.

We update our guess for the wage as follows, using Equation (10):

$$W_t(k+1) = \left(\frac{1}{\sigma} - 1 \right) A_t(k) \Pi_t(k) \tilde{I} + \frac{\chi}{\sigma} A_t(k) \Pi_t(k) (\bar{I} - \tilde{I}) + \bar{W}_t$$

We set our initial guess for the nominal wage as $W_t(0) = \bar{W}_t$, which corresponds to the value of w_{nt} when $\sigma = 1$ and $\chi = 0$. We stop at $k = K_t$ when $A_t(K_t)$ has converged and denote $A_t = A_t(K_t)$, $H_t = H_t(K_t)$ and $\Pi_t = \Pi_t(K_t)$.

We compute

$$M_t = \frac{1}{1 + \kappa_b + \kappa_w} \left([1 + \kappa_b + \kappa_w] X' - \kappa_b A_t \Pi_t \tilde{I} - \kappa_w A_t \Pi_t \bar{I} - A_t \right)$$

where we used Equation (15).

F Additional numerical results

Existence of booms and busts Figure 6 shows how σ_θ affects the conditions under which booms and busts appear. A priori, it is not clear whether σ_θ makes booms and busts more or less likely. A higher σ_θ lowers the first-period response of output to the firms' exogenous signal, and hence makes the endogenous signal respond less negatively to aggregate noise if strategic substitutability dominates, and less positively if strategic complementarity dominates. In the former case, it reduces the likelihood of booms and busts, and increases it in the former. If σ_θ is one order of magnitude larger than in the benchmark (panels (c) and (d)), the results are barely changed. If σ_θ is one order of magnitude smaller (panels (a) and (b)), then, for identical κ_a and κ_b , we need the ratio $\sigma_\lambda/\sigma_\Lambda$ to be larger. This is

especially true for conservative calibrations, and less so for liberal ones, and does not apply at all for calibrations that yield a positive κ_b .

The same reasoning applies for the role of σ_m . Figure 7 shows that if σ_m is one order of magnitude larger (panels (c) and (d)), the conditions for booms and busts are similar to the benchmark. If σ_m is one order of magnitude smaller (panels (a) and (b)), then the conditions become less stringent, especially for very negative κ_b s. Indeed, a lower σ_m triggers a milder reaction to the exogenous signal in the first period, which reduces the positive reaction of the endogenous signal in the case where strategic complementarity dominates, which generates more favorable conditions for booms and busts.

Similarly, Figure 8 shows that a smaller non-aggregate noise $\sigma_\lambda + \sigma_\Lambda$ requires slightly larger ratios $\sigma_\lambda/\sigma_\Lambda$ relative to the benchmark for booms and busts to appear (panels (a) and (b)). A larger $\sigma_\lambda + \sigma_\Lambda$, on the opposite, relaxes the conditions for booms and busts to appear (panels (c) and (d)).

The role of the wage signal Figures 9 and 10 show how the preference parameters σ and χ affect the result, for different values of σ_m . In Figure 9, we can see that, as in the benchmark, for equal values of κ_a and κ_b , changing σ does not affect the second-period output in a significant way, whether σ_m is one order of magnitude larger or lower than in the benchmark. For large value of σ_m , the wage signal is not used by firms to infer ψ because it is too noisy and is not a reliable signal of ψ . For low values of σ_m , on the opposite, the wage is a good signal of ψ , so firms are not confused by θ and booms and busts are negligible anyways.

In Figure 10, we can see that for larger values of σ_m , χ has little influence on the outcome. For lower values of σ_m , booms and busts typically do not appear for low values of χ , because the wage is a good signal. However, for larger χ s, booms and busts arise again. This is because, as apparent from Equation (14), as χ becomes larger, the wage signal is more affected by sector-specific shocks, which are represented by the sectoral noise shocks Λ_n in our set-up. These sectoral shocks then make the wage signal more noisy and less revealing of ψ , hence favoring the occurrence of booms and busts.

Contribution to fluctuations Figure 11 represents the effect of θ and ψ shocks, scaled by their standard deviation. The maximum contribution of θ happens for values of σ_θ that are intermediate (around 0.01). As σ_θ gets larger, a unitary shock

of θ leads to a larger decline in output, but then a typical shock is also smaller. Therefore, when the shock is normalized by σ_θ , its effect is smaller. As a result, in the first period, θ can contribute to up to 40% of output fluctuations. In the second period, it can contribute to up to 9% of fluctuations.

References

- Blinder, A., Canetti E. Lebow D. & Rudd J. (1998), *Asking About Prices: A New Approach to Understanding Price Stickiness*.
- Mankiw, N. Gregory and Ricardo Reis (2002), 'Sticky Information Versus Sticky Prices: A Proposal To Replace The New Keynesian Phillips Curve', *The Quarterly Journal of Economics* **117**(4), 1295–1328.

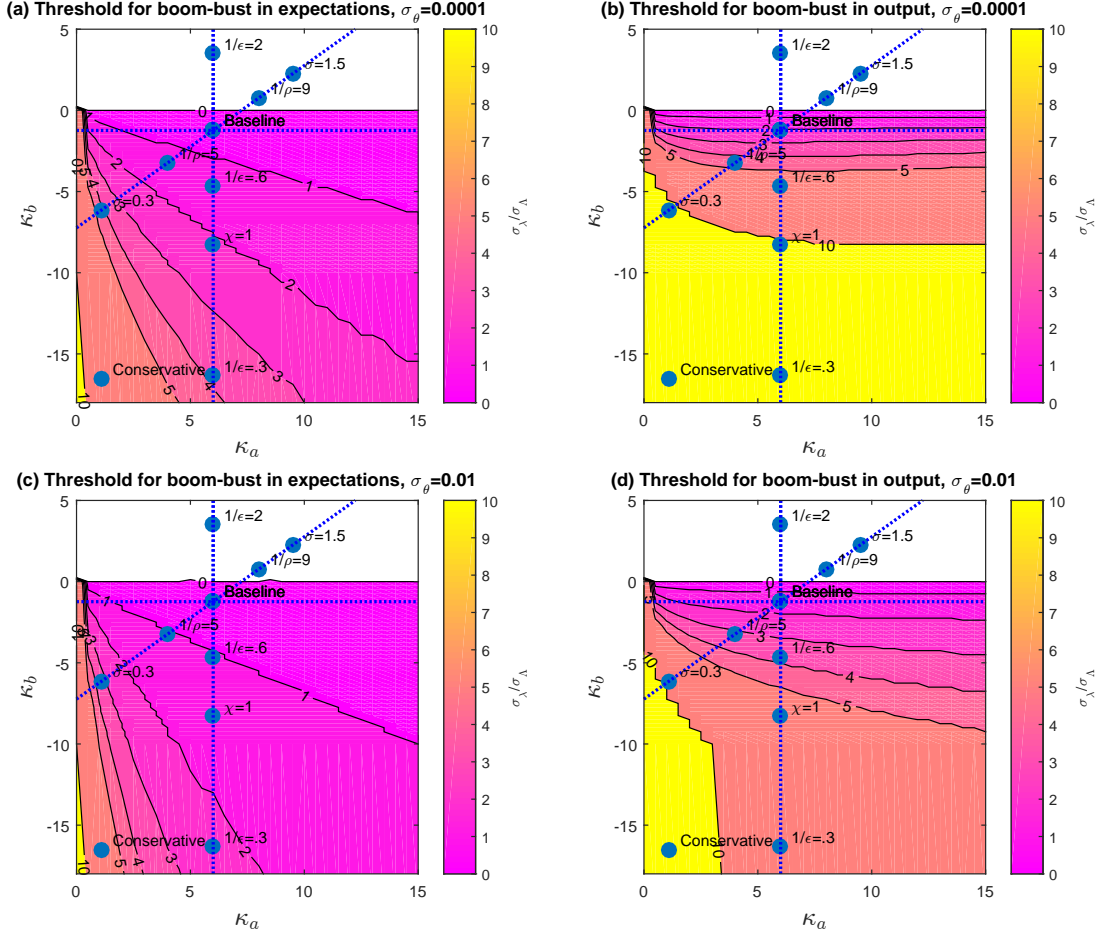


Figure 6: Thresholds for boom-bust in expectations and output - Role of σ_θ

Each curve represents threshold values for κ_b for a given ratio of $\sigma_\chi/\sigma_\lambda$. Points that are at the north-east of the curves feature booms and busts in output. $\sigma_\Lambda + \sigma_\lambda = 0.011$, σ_ψ and σ_m are set as in the benchmark calibration (see Table 1).

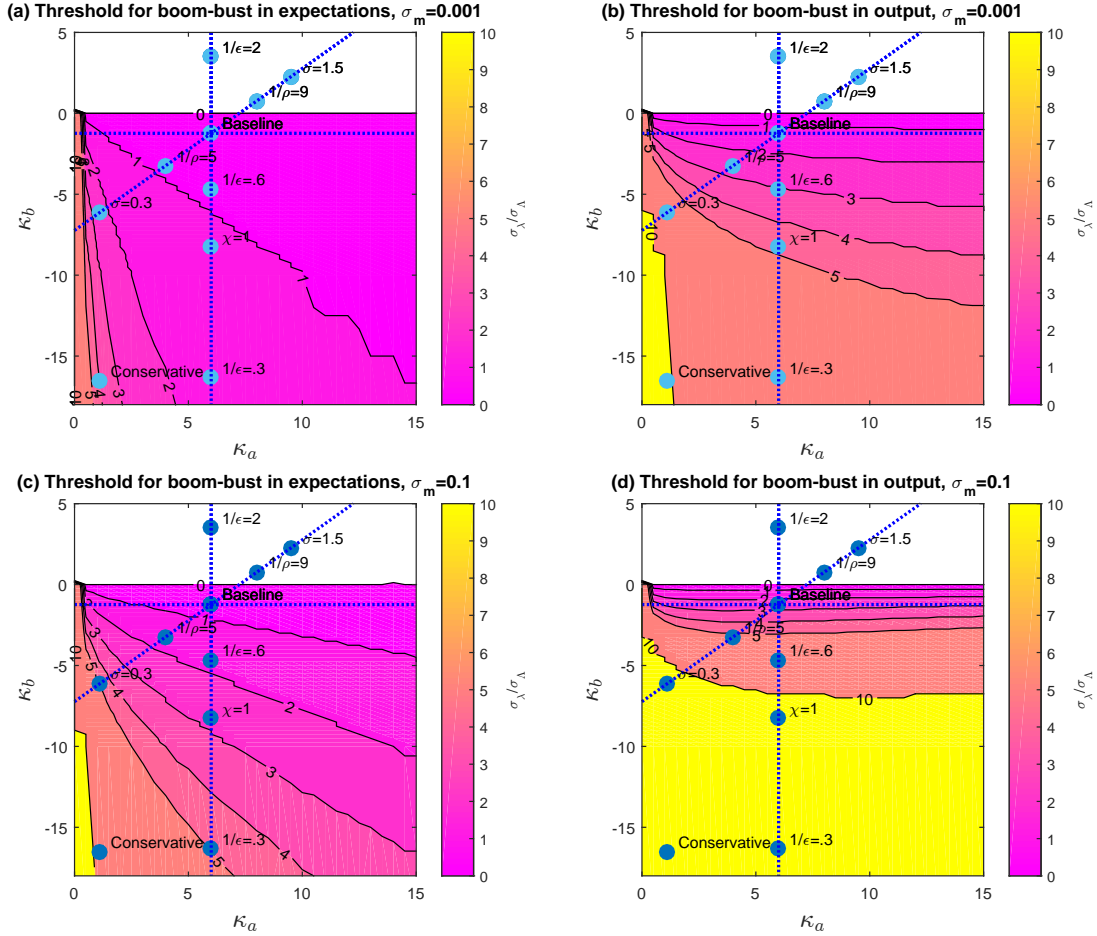


Figure 7: Thresholds for boom-bust in expectations and output - Role of σ_m

Each curve represents threshold values for κ_b for a given ratio of $\sigma_\lambda/\sigma_\Lambda$. Points that are at the north-east of the curves feature booms and busts in output. $\sigma_\Lambda + \sigma_\lambda = 0.011$, σ_ψ and σ_θ are set as in the benchmark calibration (see Table 1).

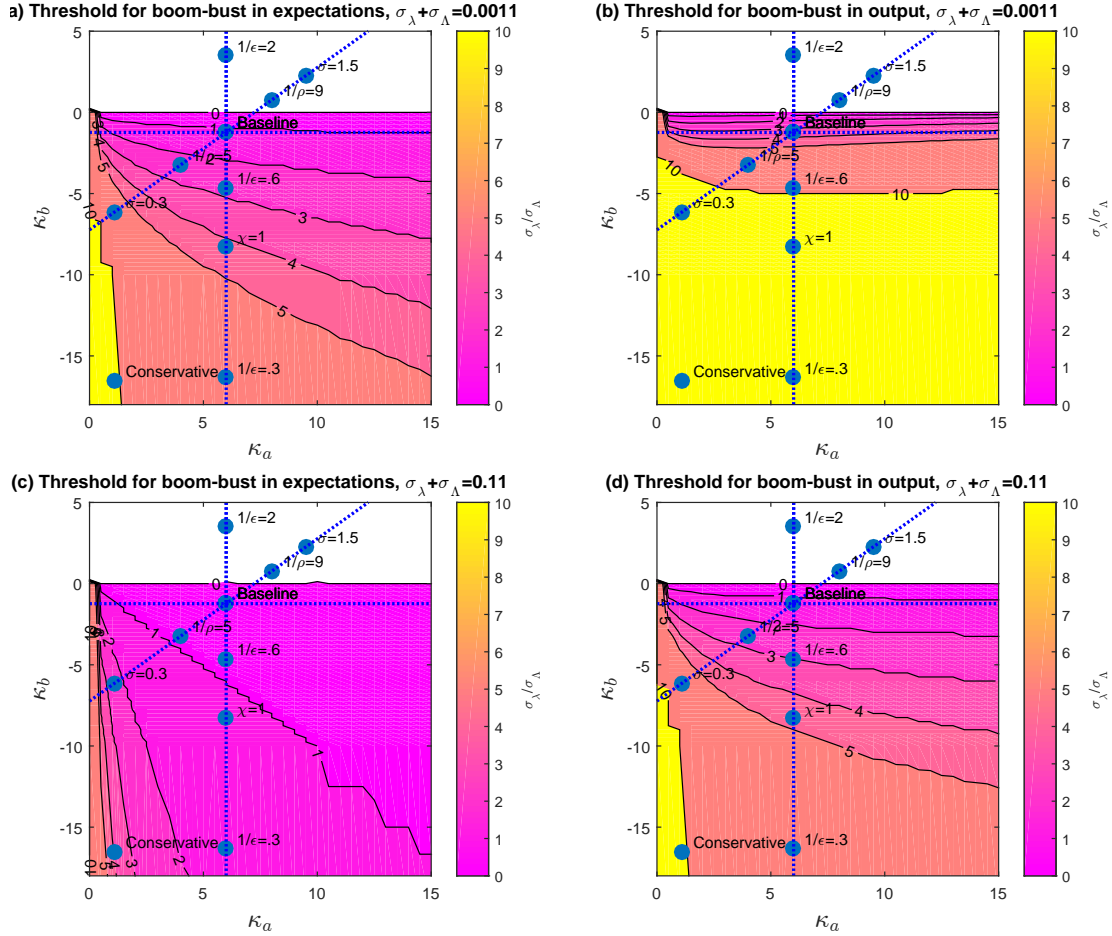


Figure 8: Thresholds for boom-bust in expectations and output - Role of $\sigma_\lambda + \sigma_\Lambda$

Each curve represents threshold values for κ_b for a given ratio of $\sigma_\lambda/\sigma_\Lambda$. Points that are at the north-east of the curves feature booms and busts in output. σ_ψ , σ_m and σ_θ are set as in the benchmark calibration (see Table 1).

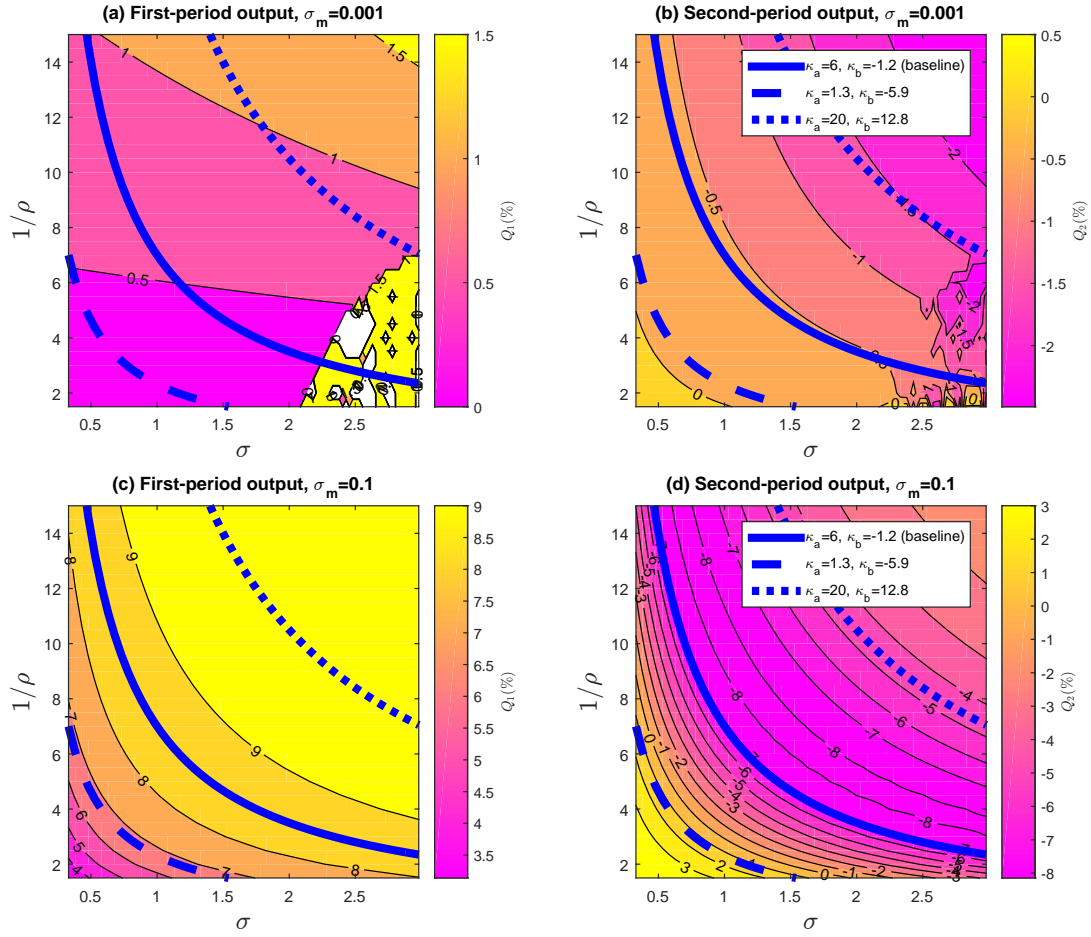


Figure 9: Role of the wage signal for alternative σ_m

We represent the effects of $\theta = 0.1$. σ_λ , σ_Λ , σ_ψ , σ_θ , χ and ϵ/ρ are set as in the benchmark calibration (see Table 1).

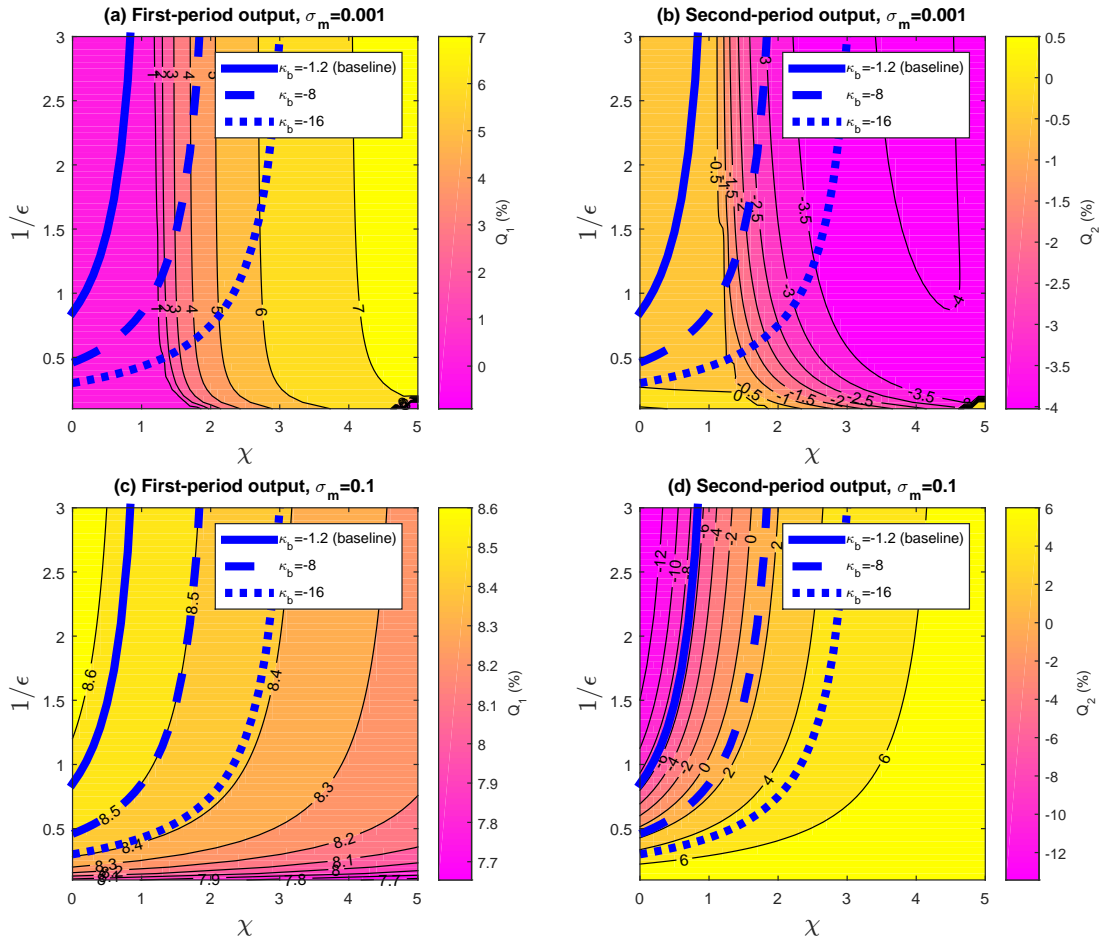


Figure 10: Role of the wage signal for alternative σ_m - continued

We represent the effects of $\theta = 0.1$. σ_λ , σ_Λ , σ_ψ , σ_θ , σ and $1/\rho$ are set as in the benchmark calibration (see Table 1).

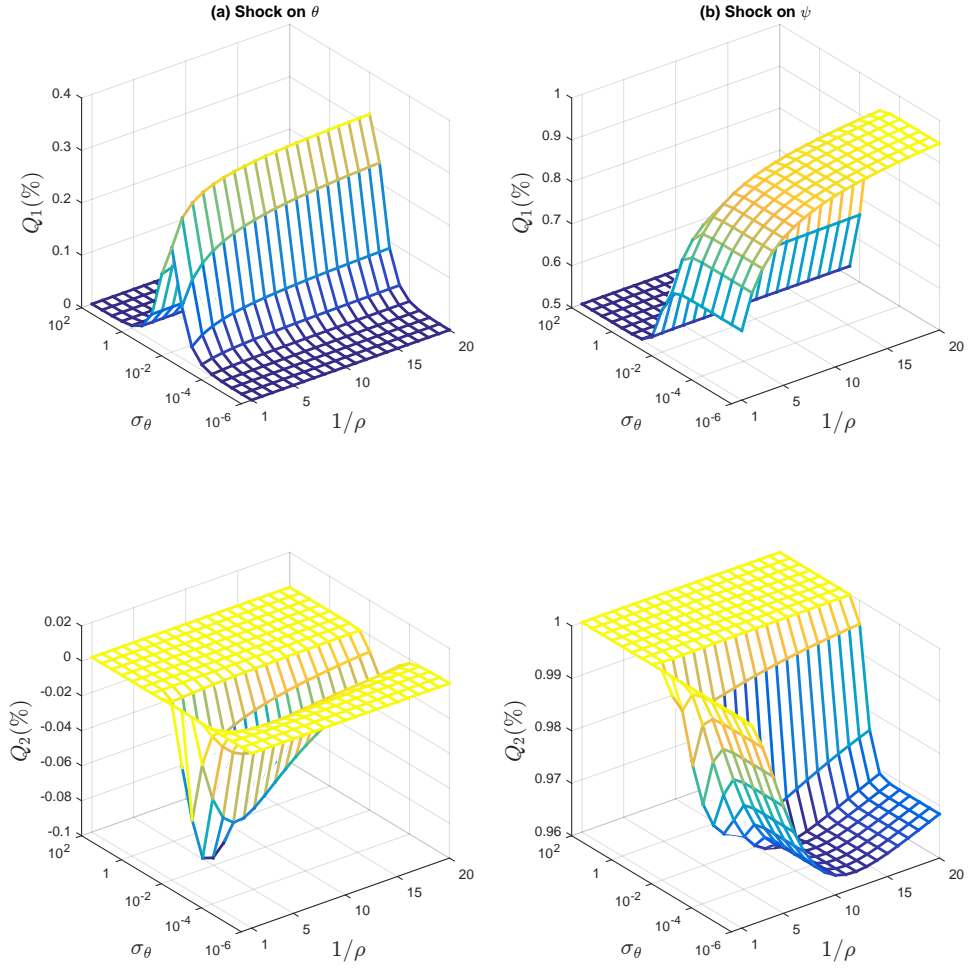


Figure 11: Effect of θ and ψ - Scaled shocks

We represent the effects of $\theta = \sigma_\theta$ (left panels) and $\psi = \sigma_\psi$ (right panels). σ_λ , σ_Λ , σ_ψ , σ_θ , σ and $1/\rho$ are set as in the benchmark calibration (see Table 1).