Optimal Monetary Policy when Information is Market-Generated

ONLINE APPENDIX

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A Extensions

A.1 Price target

We have assumed so far that the central bank actions were not conditional on any endogenous information, which does not correspond to how monetary policy is implemented in practice. We assume here that instead of observing a noisy signal of the real shock, the central bank observes the price level with noise, which is a more realistic assumption. It is also a natural way to introduce central bank noise. We then examine how optimal policy would translate into a standard Taylor-type rule, where money supply reacts to its measure of the price level. We find that the signalling policy described in Section 4 is still optimal. As in our baseline model, the central bank targets a positive correlation between money supply and prices.

Denote by ξ_p the monetary policy noise, so that the central bank observes $p-\xi_p$, with $\xi_p \sim \mathcal{N}(0, \sigma^2_{\xi_p})$.

The monetary policy rule is $m = -\beta_p(p - \xi_p)$. A positive β_p would correspond to a standard price-stabilization policy. Nominal demand then follows:

$$q = -\beta_p p + \beta_p \xi_p + v = -\beta_p p + \nu_p \tag{27}$$

where $\nu_p = \beta_p \xi_p + v$ is the total demand disturbance. Now denote $\tilde{z}_p = z + \kappa_p^{-1} \nu_p$. The precision of the signal is given by $P_p(\beta_p) = [\kappa_p(\beta_p)]^2(\sigma_v^2 + \beta_p^2 \sigma^2_{\xi_p})^{-1}$.

The analysis remains similar to our baseline. To see this, consider the endogenous signal $\tilde{y}$:

$$\tilde{y} = q + (\varrho - 1)p = (\varrho - 1 - \beta_p)p + \beta_p \xi_p + v$$

As p respond to the real shock z, the central bank can make the endogenous signal more sensitive to it by setting a large β_p in absolute value. However, as the endogenous signal reacts positively to both prices and nominal demand, it is more efficient to make nominal demand react in the same way as prices. This is achieved by setting a negative β_p.

We show indeed (see Section B.12 for details), that in equilibrium,

$$\kappa_p = \frac{(1-\chi)\delta\gamma\beta_p}{1-\gamma[\varrho(1-\chi)-1]} - \lambda \tag{28}$$

where λ is defined as in (21). As before, κ_p, the sensitivity of the endogenous signal to z, depends on the policy-induced response of nominal demand and on the natural response of prices. The natural response is the same as before (λ), while now the reaction of the nominal demand is a function of β_p but also of the natural response of prices, because policy reacts to prices and not directly to the shocks.

Again, optimal policy maximizes the precision of the endogenous signal. We show that this is achieved by setting $\beta_p = \beta_p^*$, where

$$\beta_p^* = \frac{-\sigma_v^2}{(\varrho-1)\sigma_{\xi_p}^2} \tag{29}$$

The optimal β_p still depends on the relative variance of v and ξ_p. The only difference with the baseline case is that it does not depend on λ but only on $\varrho - 1$, the elasticity of the endogenous signal to prices.

It is optimal to make nominal demand respond positively to the central bank's signal on prices. This gives more information on the real shock z to prise-setters, through the following mechanism. Prices react negatively to z, because firms receive private signals on z. As a result, the central bank measures a decrease in prices, and reacts by decreasing money supply. The endogenous signal then decreases through two channels. First, because of the price decrease, and second, because of the policy-induced decrease in nominal demand. This decrease in endogenous signal constitutes a positive signal on z, which leads firms to set even lower prices, etc... This policy design thus enables firms to react more accurately to the real shock z.

The optimal response of the money supply to the price level measure is therefore positive to emphasize the response of the endogenous signal to the real shock. The response of prices to a supply shock tends to be negative, so the optimal rule makes the money supply respond negatively to a supply shock, which would make monetary policy counter-cyclical, as in our baseline model.

A.2 Dynamic extension

Here we examine how our simple static framework can be extended to a dynamic one. The purpose is not to provide a full-fledged dynamic extension, but to show how such a dynamic model could be developed. We therefore present the simplest dynamic extension possible, and show that it can be mapped into our simple static framework, with some slight changes.

We keep the same structure as in our baseline model, and consider now that time is infinite and discreet. We also now focus on a cashless economy, as defined by Woodford (2003), where monetary policy is defined in terms of nominal interest rate, and money is not introduced explicitly.

The household is infinitely lived and has the following lifetime utility:

$$U_t = \sum_{s=0}^{\infty} \beta^s u(Y_{t+s}, N_{t+s}, Z_{t+s}) \tag{30}$$

where u, Y, N and Z are defined as in (1). The household has the same budget constraint (2) as before, except that she also holds nominal bonds B that yield the nominal interest rate i. We also introduce transfers T from the government. We thus have

$$\int_0^1 P_{it} C_{it} di + B_t = \int_0^1 \Pi_{it} di + W_t N_t + (1 + i_{t-1}) B_{t-1} + T_t$$

The budget constraint of the government being $B_{t+1} = (1 + i_t) B_t + T_t$, the resource constraint of the economy boils down to Equation (2).

The central bank sets the nominal policy rate i_t^{cb}, and the effective interest rate faced by the household is then

$$i_t = i_t^{cb} - v_t \tag{31}$$

v_t is a nominal interest rate shock that is not under the control of the central bank. It plays the same role as the velocity shock in our baseline model. The monetary policy rule is now defined as

$$i_t^{cb} = -\beta z^{cb} \tag{32}$$

The objective of the central bank is to maximize the household's utility U_t as defined in Equation (30).

For simplicity, we assume that $z = \log(Z)$ and v are i.i.d. shocks and follow the same distribution as before. Regarding information, all past shocks and past variables are known to all agents. The dynamic problem then reduces to a repeated static problem, as in our baseline. The information about the current realization of shocks follows the same structure as in our baseline model. The objective of the central bank then boils down to minimizing the same loss function as in the baseline model. All our key equations hold, especially the pricing equation (6). The endogenous signal is still $\tilde{y} = y + \varrho p$. We define nominal spending as before $q = y + p$, so that $\tilde{y} = q + (\varrho - 1)p$.

The only difference with our static model is the way nominal demand q is determined. From the model's Euler equation, we can show that

$$(\phi y_t + p_t) = E_t(\phi y_{t+1} + p_{t+1}) - i_t$$

where $E_t(.)$ represents the household's expectations. Since all our shocks are i.i.d., $E_t(y_{t+1}) = E_t(p_{t+1}) = 0$, which enables us to write, using $q = y + p$, and the monetary policy rule (32), along with the interest rate equation (31):

$$q_t = \frac{1}{\phi}(\beta z^{cb} + v_t) - \frac{1-\phi}{\phi} p_t = \frac{1}{\phi}(\beta z + \nu_t) - \frac{1-\phi}{\phi} p_t \tag{33}$$

where $\nu = \beta\xi + v$ as before. This equation is the counterpart of Equation (12). Nominal demand is related to the policy rule through the elasticity of intertemporal substitution $1/\phi$. Importantly, nominal demand does not depend on policy only. The price now plays a role as well, as a higher price today means a higher expected real interest rate, which has a negative impact on demand.

We can show that the optimal β is still the one that maximizes the precision of the endogenous signal (see Section B.13). It is now defined as follows:

$$\beta^* = -\frac{\sigma_v^2}{\lambda \sigma_\xi^2} \tag{34}$$

where λ is given by

$$\lambda = \frac{(\phi\varrho - 1)(1-\chi)\gamma\delta}{1 + [\phi\varrho(1-\chi) - 1]\gamma} \tag{35}$$

Notice that now, the sign of λ can be negative, depending on the sign of $\phi\varrho - 1$. This comes from the fact that the effect of aggregate prices on the endogenous signal is now ambiguous. On the one hand, higher prices increase the demand for the individual good, for a given level of nominal demand. This effect was present in the static model and is governed by the elasticity of substitution ϱ. However, now higher prices decrease nominal demand, which decreases the demand for the individual good. This effect was absent in the static model and is governed by the elasticity of intertemporal substitution $1/\phi$. When $\phi\varrho < 1$, the latter effect dominates, which is reflected in a negative λ.

In that case, it would be optimal to set a positive β. Indeed, a positive supply shock, by decreasing prices, will have a positive effect on the endogenous signal. An increase in the endogenous signal is therefore good news about z for firms. When the central bank itself gets a good news about z, it can reinforce the information content of the endogenous signal by decreasing the nominal interest rate and hence further stimulating nominal demand, which will reinforce the positive effect of z on the endogenous signal.

Can $\phi\varrho$ be lower than 1 for realistic parameter values? The admissible values for $1/\phi$ go from 0.1 to 2.[1] The value for the elasticity of substitution between goods ϱ has been estimated in the range of 1.5-10, where lower values correspond to estimations on aggregate data and higher values to estimations on sectoral data.[2] Whereas we cannot exclude this case completely, $\phi\varrho$ is lower than 1 only for extremely low values of ϱ, and extremely large values of $1/\phi$. Our main predictions are therefore most likely to carry through in this simple dynamic framework.

Discussion How could the dynamic analysis be extended further and how would that change our results? First, more persistence in shocks could be added. With more persistent shocks, and under the assumption that past shocks are not known, it would be relevant to make the policy rate depend on past signals as well, which would give rise to some interest rate smoothing.

Second, the structure of the model could be enriched. In particular, what appears to be crucial is the structure of the endogenous signal, and especially, how that signal depends respectively on aggregate prices and on policy-related demand. In our baseline model, aggregate prices affect positively the endogenous signal. Since the real shock has a negative effect on prices, this led to a counter-cyclical monetary policy. If aggregate prices affected the endogenous signal negatively, then the signalling policy would be pro-cyclical. This result could be obtained if prices had a sufficiently negative effect on aggregate demand – and hence on the individual demand for good. The introduction of a Fisherian debt-deflation mechanism, for instance, because it makes aggregate demand react more negatively to the price

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[1] See for instance Hall (1988), Barro (2009), and, for a meta-analysis, Havranek et al. (2015).

[2] See Imbs and Méjean (2015).

level, could make a pro-cyclical policy a better way to reveal information to agents.

A.3 Signal on the nominal shock

In our baseline model, we have assumed that the central bank and firms received information only on the real shock (z), but not on the nominal or velocity shock (v). We relax this assumption here, by assuming that both the central bank and firms receive noisy signals on v.

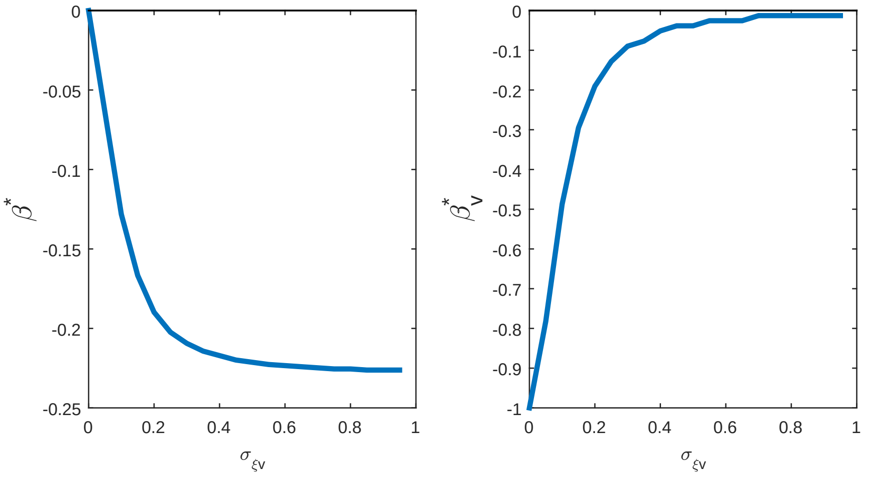
We show that, while the central bank effectively emphasizes the price response to real shocks, it stabilizes prices as a response to nominal shocks. As the nominal shock is inflationary, the central bank reduces money supply following a positive signal on the nominal shock, in order to limit its impact on the endogenous signal, which stabilizes the response of price to the nominal shock. The objective of the central bank is still to make the endogenous signal the best signal possible of the real shock. It has therefore to minimize the impact of nominal shocks on the endogenous signal. This is an implication of the divine coincidence through which a better information on the real shock implies a better information on nominal demand.

We assume that the central bank gets a noisy signal on v: $v^{cb} = v + \xi_v$, where $\xi_v \sim \mathcal{N}(0, \sigma^2_{\xi_v})$. Each firm also receives a private signal on v: $v_i = v + \epsilon_{vi}$, where $\epsilon_{vi} \sim \mathcal{N}(0, \sigma^2_{\epsilon_v})$ is an idiosyncratic shock.

Figure (3) represents the optimal β and β_v under different values of σ_{ξ_v}. The optimal response to both real and nominal central bank signals is negative. As the real shock is inflationary while the nominal shock is deflationary, this implies that it is optimal to make money supply comove positively with prices in the case of real shocks, as in the baseline model, and negatively in the case of nominal shocks, whatever the level of σ_{ϵ_v} and σ_{ξ_v}.

Consider now the effect of σ_{ξ_v}. When the central bank is perfectly informed on v (σ_{ξ_v} goes to 0), β^*_v goes to -1 and β^* goes to zero: it is enough to shut down nominal shocks to make the endogenous variable reveal z to firms perfectly. As the central bank becomes less informed on v, (σ_{ξ_v} increases), then it reacts less and less to its signal on v (β^*_v is less negative) and more and more to its signal on z (β^* is more negative).

Figure 3: Optimal policy with information on the nominal shock



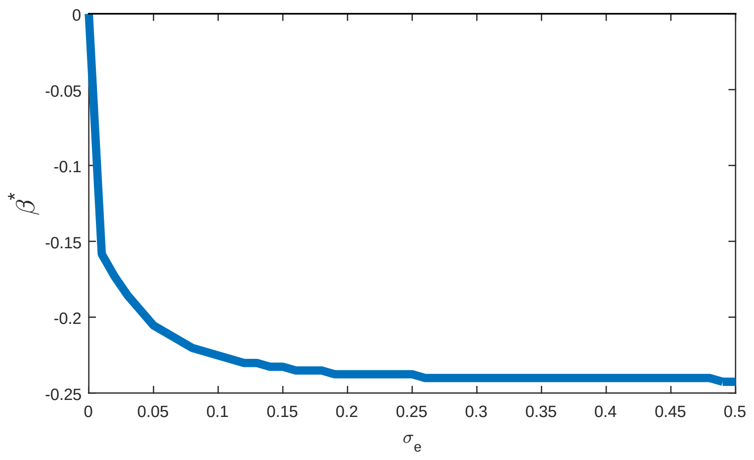
Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set in the baseline $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$, $\sigma_{\epsilon_v} = 1$ and $\sigma_\xi = 0.2$.

A.4 Marginal cost signal

The firms' information problem is to infer their marginal cost. In our simple framework, this marginal cost corresponds to the wage, which is observed only at the end of period. If the wage were known at the price-setting stage, then the signal extraction problem of firms would become trivial, as it would be optimal to simply set $p_i = w$. As argued earlier, in our view, it is reasonable to assume that firms do not observe their marginal cost directly. However, it is also reasonable to assume that firms have some information on their marginal cost. We introduce this idea by assuming that firms observe a signal on the wage $w_i = w + e_i$, where $e_i \sim \mathcal{N}(0, \sigma_e^2)$ is an idiosyncratic noise. We then examine how this assumption affects the equilibrium outcome and optimal policy.

This extension is solved numerically, with different levels of σ_e. The results are shown in Figure 4. The optimal β is still negative in all cases, which means that our signalling channel is still present. The accuracy of the marginal cost signal decreases the magnitude of the optimal β though: it appears to get closer to zero as the level of noise σ_e diminishes. A greater accuracy of the marginal cost signal has the same effect as a greater accuracy of the private signal on z (a lower σ_ϵ, i.e. a higher γ). A greater accuracy of private information in general makes the

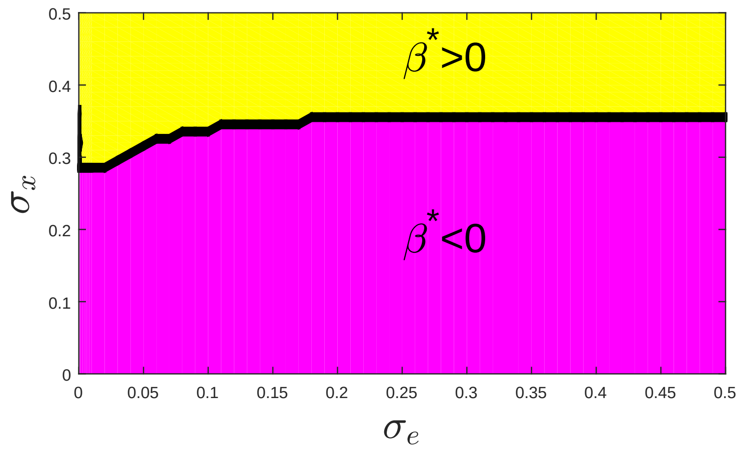
Figure 4: Optimal policy with a marginal cost signal



Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.2$.

endogenous signal a better signal of the real shock, which renders central bank intervention less necessary.

Figure 5: Optimal policy with noisy marginal cost and demand signals



Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.2$.

Figure 4 considers the special case where the demand signal is perfectly observed, so that the optimal policy is necessarily the signalling policy. We now relax this assumption by considering varying levels of noise in the demand signal σ_x, along with varying levels of noise in the marginal cost signal σ_e. Figure 5 displays the sign of β^* for different values of σ_x and σ_e. We can then see what channel prevails (signalling or surprise) depending on the relative levels of uncertainty in demand and uncertainty in marginal cost. Note that *a priori*, it is not clear how uncertainty on the marginal cost should affect the dominant channel, for a given uncertainty on demand. Intuitively, a better marginal cost signal helps firms set prices more accurately (since at the optimum the price is equal to the marginal cost). For a given quality of information on demand, better information on the marginal cost thus renders costly monetary policy interventions less desirable, regardless of the channel. The figure actually shows that the sign of β^* depends essentially on the precision of the demand signal σ_x. That is, the signalling channel vanishes when the demand signal is too noisy, which we have already shown in Section 4. The precision of the marginal cost signal plays almost no role in determining the threshold σ_x above which β^* becomes positive. Only when the marginal cost signal is particularly precise (when σ_e is below 0.2) does σ_e play a role. In that case, the surprise channel starts prevailing for relatively lower values of σ_x, that is, for relatively more precise demand signals.

A.5 "Inefficient" shocks

We introduce shocks to the elasticity of substitution between goods ϱ, which is now time-varying. Mark-ups are inversely related to ϱ, so we denote ρ the opposite of the log-deviation of ϱ from its steady state, and interpret ρ as shocks to the mark-up. By introducing mark-up shocks, we introduce shocks that drive inefficient fluctuations. In that case, the central bank does not necessarily want to improve the information of firms.

We assume that $\rho \sim \mathcal{N}(0, \sigma_\rho^2)$. The optimal price-setting equation now becomes

$$p_i = \chi E_i p + (1 - \chi)[E_i q + \delta_\rho E_i \rho], \tag{36}$$

with $\delta_\rho = 1/[(\eta + \phi)(\varrho - 1)]$. The price responds to the firm's expectation of the mark-up shock. Notice that δ_ρ is positive, which means that a mark-up shock tends to be inflationary, contrary to the supply shock, which is deflationary.

Firms have the same information set as before, but they also receive an individual signal ρ_i on ρ: $\rho_i = \rho + \omega_i$, where $\omega_i \sim \mathcal{N}(0, \sigma_\omega^2)$ is an idiosyncratic shock.

The central bank observes a signal ρ^{cb} on the mark-up shock ρ: $\rho^{cb} = \rho + \xi_\rho$ where $\xi_\rho \sim \mathcal{N}(0, \sigma_{\xi_\rho}^2)$. It then sets money supply as $m = \beta_\rho \rho^{cb}$, so that the nominal demand becomes

$$q = \beta_\rho \rho + \nu_\rho,$$

with $\nu_\rho = v + \beta_\rho \xi_\rho$.

All the analysis of Section 3 holds, except for the definition of the price gap $p - p^*$ and for Proposition 1 which defined optimal policy. Indeed, since mark-up shocks drive inefficient output fluctuations, the socially optimal price is $p^* = q$. The spread between the realized and optimal price is then

$$p_i - p^* = \chi[E_i(p) - p^*] + (1 - \chi)\{[E_i(q) - q] + \delta_\rho[E_i(\rho) - \rho]\} + \delta_\rho \rho \tag{37}$$

As before, the expectation errors can be reduced by making the endogenous signal as informative as possible. However, this would not be necessarily optimal here because it would make agents respond better to mark-up shocks, which would drive inefficient fluctuations. Indeed, the price gap would still be equal to $\delta_\rho \rho$ if the firms made no expectation errors. It could then be optimal for the central bank to make the endogenous signal less precise.

In fact, the optimal monetary policy *minimizes* the precision. The optimal policy is then given by β_ρ^* defined as follows (the proof is available in Section B.14):

$$\beta_\rho^* = -\lambda_\rho(\beta_\rho^*) = \frac{-(\varrho - 1)(1 - \chi)\delta_\rho \sigma_\omega^{-2}}{\sigma_\rho^{-2} + \varrho(1 - \chi)\sigma_\omega^{-2}} \tag{38}$$

Now, the purpose of the central bank is to reduce the information content of the endogenous signal by counteracting the effect of mark-up shocks on prices. Since a mark-up shock is inflationary, monetary policy is restrictive following a mark-up shock, so that the endogenous signal does not reveal anything about that shock to the agents. This contributes to lower the responsiveness of output to the inefficient mark-up shocks.[3]

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[3] Our results are consistent with Angeletos and Pavan (2007) and Angeletos et al. (2016). They find that when the business cycle is driven by non-distortionary forces (e.g., productivity shocks), welfare always increases with information's precision. When the business cycle is driven by distortionary forces (e.g., mark-up shocks), welfare decreases with information.

B Proofs

B.1 The Price-setting Equation

The profits of firm i are $\Pi_i = P_i Y_i - W N_i$. Using the individual good demand equation, (3), the production technology, (5), we can write these profits as

$$\Pi_i = (P_i - W) Y \left(\frac{P_i}{P} \right)^{-\varrho}$$

The optimality condition with respect to P_i, resulting from the model specified in section 2 is then

$$E_i \left\{ Y \left(\frac{P_i}{P} \right)^{-\varrho} \right\} = E_i \left\{ \varrho \frac{(P_i - W)Y}{P} \left(\frac{P_i}{P} \right)^{-(\varrho+1)} \right\}. \tag{39}$$

We log-linearize (39) around the non-stochastic steady-state characterized by $Y = \bar{Y}$, $P = \bar{P}$, $P_i = \bar{P}_i = \bar{P}$, $W = \bar{W}$, and obtain

$$\bar{Y} E_i \left\{ y - \varrho(p_i - p) - \bar{y} \right\} = \varrho \bar{Y} E_i \left\{ y - \varrho(p_i - p) - \bar{y} - \frac{\bar{W}}{\bar{P}} \left[y - \varrho(p_i - p) + w - p_i - \bar{y} - (\bar{w} - \bar{p}) \right] \right\},$$

which yields

$$\varrho \frac{\bar{W}}{\bar{P}} E_i \left\{ y - \varrho(p_i - p) + w - p_i - \bar{y} - (\bar{w} - \bar{p}) \right\} = (\varrho - 1) E_i \left\{ y - \varrho(p_i - p) - \bar{y} \right\}.$$

Using $\frac{\bar{W}}{\bar{P}} = \frac{\varrho - 1}{\varrho}$ in the non-stochastic steady state, we get

$$\varrho e^{\bar{w} - \bar{p}} E_i \left\{ w - p_i - (\bar{w} - \bar{p}) \right\} = 0.$$

Using $\bar{w} - \bar{p} = \log[(\varrho - 1)/\varrho]$, we find that the log-linear version of equation (39) is:

$$p_i = E_i(w) + \log[1 + 1/(\varrho - 1)].$$

In order to find w, we use the household's optimality conditions and equation (3)

$$\frac{W}{P} = \frac{N^\eta}{Z Y^{-\phi}},$$

which yields in log-linear terms:

$$w = p + \eta n + \phi y - z.$$

Using the log-linear approximations $y = \int_0^1 y_i di$, and $n = \int_0^1 n_i di$, we get $n = y$ as $y_i = n_i$, which implies that the individual price is set as follows:

$$p_i = E_i(p) + (\eta + \phi)E_i(y) - E_i(z),$$

where we discard constant terms.

Finally, replacing y with $q - p$, we obtain the price-setting equation (6).

B.2 The Welfare Approximation

The efficient level of output is defined as the one that emerges under a social planner in the symmetric equilibrium, where $P_i = P$, $W = P$ and $Y_i = Y$ for all i. In that case, the marginal rate of substitution between labour and consumption is equal to the marginal product of labour. With $N = Y$, this gives

$$Y^{\phi+\eta} = Z$$

that in logs becomes

$$y = \frac{1}{\phi + \eta} z. \tag{40}$$

Efficient output is therefore $y^* = \delta z$. Besides, (40) should be valid in the steady state as well so $\bar{y} = \frac{1}{\phi+\eta}\bar{z}$.

Using the full employment condition and the production function for each individual good i, we can rewrite the utility function as

$$u(Y, N, Z) = \frac{e^{(1-\phi)y}}{1-\phi} - e^{-z}\frac{\left(\int_0^1 e^{y_i} di\right)^{1+\eta}}{1+\eta}. \tag{41}$$

Note that $y_i = y + y_i - y$. The second-order approximation of the utility function is then

$$\begin{aligned} u(Y, N, Z) \approx\ & e^{(1-\phi)\bar{y}}\left(y + \tfrac{1-\phi}{2}y^2\right) \\ & -e^{(1+\eta)\bar{y}-\bar{z}}\left(y + \tfrac{1+\eta}{2}y^2 + \int_0^1 (y_i - y)di + \int_0^1 (y_i - y)^2 di - zy - z\int_0^1 (y_i - y)di\right) \\ & +t.i.p.,, \end{aligned} \tag{42}$$

where *t.i.p.* stands for "terms that are independent of policy" and includes the terms that are completely exogenous, like productivity.

Defining the cross sectional mean as $E_i(y_i) = \int_0^1 y_i di$, the cross sectional variance as $Var_i(y_i - y) = \int_0^1 (y_i - y)^2 di$, and taking into account that $(1-\phi)\bar{y} = (1+\eta)\bar{y} - \bar{z}$, Equation (42) becomes

$$u(Y,N,Z) \approx e^{(1-\phi)\bar{y}} \left(-\frac{\eta+\phi}{2} y^2 - [E_i(y_i) - y] - Var_i(y_i - y) + zy + zVar_i(y_i - y) \right) + t.i.p.,, \tag{43}$$

Using the second-order approximation of the consumption bundle

$$y = E_i(y_i) + \frac{1-\varrho^{-1}}{2} Var_i(y_i - y) \tag{44}$$

we can rewrite the approximation in (43) as

$$\begin{aligned} u(Y,N,Z) &\approx -e^{(1-\phi)\bar{y}} \left(\frac{\eta+\phi}{2} y^2 - zy - \frac{1}{2\varrho} Var_i(y_i - y) - zVar_i(y_i - y) \right) + t.i.p., \\ &\approx -e^{(1-\phi)\bar{y}} \left(\frac{\eta+\phi}{2} \left[y - \frac{1}{\eta+\phi} z \right]^2 - \frac{1}{2\varrho} Var_i(y_i - y) - zVar_i(y_i - y) \right) + t.i.p. \end{aligned}$$

Given that $\delta = 1/(\phi+\eta)$ and $\chi = 1 - (\eta+\phi)$, this becomes

$$u(Y,N,Z) \approx -e^{(1-\phi)\bar{y}} \frac{\eta+\phi}{2} \left([y - \delta z]^2 - \frac{1}{\varrho(1-\chi)} Var_i(y_i - y) - 2\delta z Var_i(y_i - y) \right) + t.i.p., \tag{45}$$

Dropping the proportionality factor and substituting in the second term the full information production $y^* = \delta z$, gives:

$$\begin{aligned} -Eu(Y,N,Z) &\approx E(y-y^*)^2 + \tfrac{1}{\varrho(1-\chi)} Var_i(y_i - y) + t.i.p. \\ &\approx Var(y-y^*) + \tfrac{1}{\varrho(1-\chi)} Var_i(y_i - y) + t.i.p., \end{aligned} \tag{46}$$

Finally, note that, by defining $p^* = q - \delta z$ and by using the aggregate demand equation (7), we get $y - y^* = -(p - p^*)$. Besides, the individual demand equation (9) yields $y_i - y = -\varrho(p_i - p)$, so (46) rewrites as

$$-Eu(Y,N,Z) \approx Var(p-p^*) + \frac{\varrho}{(1-\chi)} Var_i(p_i - p) + t.i.p.$$

Then simply define $\Phi = \varrho/(1-\chi)$ to obtain the central bank loss function (11).

B.3 Proof of Lemma 1

We first solve for the equilibrium pricing equation, given our guess of the endogenous signal: $\tilde{z} = z + \kappa^{-1}\nu$, then characterize κ, and finally study existence and unicity of the solution.

Solving the pricing equation Replacing the monetary policy rule (12) in the price-setting equation (6), we obtain

$$\begin{aligned} p_i &= \chi E_i(p) + (1-\chi) E_i[(\beta-\delta)z + \nu] \\ &= \chi E_i(p) + (1-\chi) E_i[(\beta-\delta-\kappa)z + \kappa\tilde{z}] \end{aligned} \tag{47}$$

We make the following guess: $p_i = \alpha z_i + \tilde{\alpha}\tilde{z}$. This yields $p = \alpha z + \tilde{\alpha}\tilde{z}$. Taking expectations of p and z and replacing in (47), we get

$$\begin{array}{rlll} p_i = & \Big\{ \chi\alpha\gamma & +(1-\chi)(\beta-\delta-\kappa)\gamma & \Big\} z_i \\ + & \Big\{ \chi\Big[\alpha\tilde{\gamma}+\tilde{\alpha}\Big] & +(1-\chi)\Big[(\beta-\delta-\kappa)\tilde{\gamma}+\kappa\Big] & \Big\} \tilde{z} \end{array}$$

After identifying the coefficients and re-arranging we find

$$\alpha = \frac{(1-\chi)(\beta-\delta-\kappa)\gamma}{1-\chi\gamma} \qquad \tilde{\alpha} = \kappa + \frac{(\beta-\delta-\kappa)\tilde{\gamma}}{1-\chi\gamma} \tag{48}$$

Characterization Firms observe the adjusted demand $\tilde{y} = y + \varrho p = q + (\varrho - 1)p = \beta z + \nu + (\varrho-1)p = \kappa\tilde{z} + (\beta-\kappa)z + (\varrho-1)p$. Using our guess-and-verify solution for p_i, we infer that $p = \alpha z + \tilde{\alpha}\tilde{z}$ and then replace in $\tilde{y}$, so we can write $\tilde{y} = [\kappa + (\varrho-1)\tilde{\alpha}]\tilde{z} + [\beta - \kappa + (\varrho-1)\alpha]z$.

$\tilde{y}$ depends linearly on z and ν. For our guess to be true, we need that $\tilde{y} = f\tilde{z}$ for some constant f. Identifying the coefficients, we get

$$\beta - \kappa + (\varrho-1)\alpha = 0$$

replacing α using (48) and rearranging, we get (20).

Existence and unicity Equation (20) can be rewritten as $X(\kappa) = \beta$ with

$$X(\kappa) = \kappa + \frac{(\varrho-1)(1-\chi)\sigma_\varepsilon^{-2}\delta}{\sigma_z^{-2} + \kappa^2\sigma_\nu^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}} \tag{49}$$

X is continuous on $\mathbb{R}$ with values between $-\infty$ and $+\infty$, so a solution $\kappa(\beta)$ to (20) exists.

Suppose $\beta < 0$. According to (56), $\kappa < X(\kappa)$ as $\delta > 0$, so necessarily a solution $\kappa(\beta) < \beta < 0$. For the solution to be unique, it is then sufficient that X is strictly increasing for all $\kappa < 0$. X is continuously differentiable with

$$X'(\kappa) \;=\; 1 - \frac{2\kappa(\varrho-1)(1-\chi)\sigma_\varepsilon^{-2}\sigma_\nu^{-2}\delta}{[\sigma_z^{-2}+\kappa^2\sigma_\nu^{-2}+\varrho(1-\chi)\sigma_\varepsilon^{-2}]^2}$$

This is positive if $\kappa < 0$, as $\delta > 0$. So X is strictly increasing in κ for $\kappa \in \mathbb{R}-$. Therefore, for $\beta < 0$, the solution $\kappa(\beta)$ of $X(\kappa) = \beta$ is unique.

When $\beta > 0$, there could be multiple solutions.

B.4 Proof of Equations (24)

Note that we can write $p^* = (\beta - \delta)z + \nu = (\beta - \delta - \kappa)z + \kappa\tilde{z}$. Then combining with $p = \alpha z + \tilde{\alpha}\tilde{z}$, where α and $\tilde{\alpha}$ are defined by (48), we obtain

$$p - p^* = \frac{(\beta - \delta - \kappa)}{1 - \chi\gamma}[(1 - \gamma - \tilde{\gamma})z - \tilde{\gamma}\kappa^{-1}\nu]$$

Using the above expression for p_i, we can also write

$$p_i - p = \frac{(1 - \chi)(\beta - \delta - \kappa)}{1 - \chi\gamma}\gamma\varepsilon_i$$

Finally, note that

$$\begin{aligned} \bar{E}z - z &= -(1 - \gamma - \tilde{\gamma})z + \tilde{\gamma}\kappa^{-1}\nu \\ E_i z - \bar{E}z &= \gamma\varepsilon_i \end{aligned}$$

and that $\beta - \delta - \kappa = \lambda - \delta$, which yields (24).

B.5 Proof of Lemma 2

We first write more explicitly the loss function:

$$L = \delta^2 \frac{\sigma_z^{-2} + P(\beta) + \Phi(1 - \chi)^2\sigma_\varepsilon^{-2}}{[\sigma_z^{-2} + P(\beta) + \varrho(1 - \chi)\sigma_\varepsilon^{-2}]^2} \tag{50}$$

where $P(\beta) = \kappa(\beta)^2[\beta^2\sigma_\xi^2 + \sigma_v^2]^{-1}$. This expression is obtained by replacing the output gap and the individual deviations using (24), then replacing κ using (20), and finally by replacing γ, $\tilde{\gamma}$ and $\tilde{\chi}$ by their expressions.

We then replace $\Phi = \varrho/(1 - \chi)$ and obtain

$$L = \frac{\delta^2}{\sigma_z^{-2} + P(\beta) + \varrho(1 - \chi)\sigma_\varepsilon^{-2}} \tag{51}$$

L depends on β only through the precision $P(\beta)$. L decreases monotonously with the precision $P(\beta)$. Therefore, the level of β that maximizes the precision also minimizes L.

B.6 Proof of Proposition 1

Take the log of $P(\beta)$ to obtain $\log(P(\beta)) = 2\log(\kappa(\beta)) - \log(\beta^2\sigma_\xi^2 + \sigma_v^2)$. The precision has a component due to the sensitivity of the signal (first part) and a component due to the policy-induced noise (second part). The optimal β is then defined by:

$$\frac{\partial \log(P)}{\partial \beta} = \frac{2}{P(\beta)} \left[\underbrace{\frac{\beta\kappa'(\beta)}{\kappa(\beta)}}_{\text{Elasticity of } \kappa \text{ to } \beta} - \underbrace{\frac{\beta^2\sigma_\xi^2}{\beta^2\sigma_\xi^2 + \sigma_v^2}}_{\text{Share of policy-induced noise in demand disturbance}} \right] = 0 \tag{52}$$

The equilibrium β therefore equalizes the elasticity of κ to β to the share of policy-induced noise in the endogenous signal's noise.

We can show that at the optimum, the elasticity is equal to the share of policy in the signal sensitivity β/κ. To establish this result on the elasticity, suppose there exists β^* such that $\partial \log(P)(\beta^*)/\partial\beta = 0$. This implies that $\kappa'(\beta^*) = 1$, since, according to (20) and (21),

$$\kappa'(\beta) = 1 + \frac{(\varrho - 1)(1-\chi)\delta(\sigma_\varepsilon^2)^{-1}P(\beta)\partial\log(P)/\partial\beta}{[\sigma_z^{-2} + P(\beta) + \varrho(1-\chi)\sigma_\varepsilon^{-2}]^2}$$

Therefore, $\beta\kappa'(\beta)/\kappa(\beta) = \beta/\kappa$ at the optimum.

Replacing in $\partial\log(P)/\partial\beta$ and rearranging, we find that β^* must satisfy

$$\frac{\beta^*}{\kappa(\beta^*)} = \frac{(\beta^*)^2\sigma_\xi^2}{(\beta^*)^2\sigma_\xi^2 + \sigma_v^2},$$

which characterizes β^* uniquely. This equation can be rewritten using $\kappa = \beta + \lambda$ to obtain Equation (25).

B.7 The simple model as a limit case

In this paragraph we prove that the simple model with endogenous information and rational firms presented in section 3.1 of the paper can be interpreted as a limit case of the full model where $\chi = 0$, $\varrho \to 1$, $\sigma_\xi \to 0$ and $\sigma_v \to 0$. To do so, we consider the general solution of the full model, and apply $\chi = 0$, $\varrho \to 1$, $\sigma_\xi \to 0$, $\sigma_v \to 0$ and $\beta \neq 0$.

First, consider the solution for λ as expressed in Equation (21). For $\chi = 0$, we have

$$\lambda = \frac{(\varrho - 1)\gamma\delta}{1 + (\varrho - 1)\gamma}.$$

Since $0 \leq \gamma \leq 1$ and $\varrho > 1$, λ is bounded:

$$|\lambda| \leq (\varrho - 1)\delta,$$

which implies that $\lambda \to 0$ as $\varrho \to 1$. As a result, according to Equation (20), $\kappa \to \beta$.

Now, consider the endogenous demand signal defined in Equation (10) as $\tilde{y} = q + (\varrho - 1)p$. Using the guess made in Appendix B.3, $p = \alpha z + \tilde{\alpha}\tilde{z}$, we substitute our new parameter values, $\chi = 0$ and $\varrho \to 1$, in the solutions of α and $\tilde{\alpha}$, as shown in Equation (48). As a result, $\alpha \to -\delta\gamma$ and $\tilde{\alpha} \to \beta - \delta\tilde{\gamma}$. Using Equation (22), we can show that, if $\beta \neq 0$, $\tilde{z} \to \beta^{-1}q$. Consequently, $p \to -\delta\gamma z + (1 - \beta^{-1}\delta\tilde{\gamma})q$ and $\tilde{y} \to q$, which is exactly the value of the demand signal in our simple model.

Consider the normalized demand signal $\tilde{z}$, which, as we have shown, converges to $\beta^{-1}q$. Note that aggregate nominal demand can be written as $q = \beta z + \beta\sigma_\xi(\xi/\sigma_\xi) + \sigma_v(v/\sigma_v)$, with $(\xi/\sigma_\xi) \sim \mathcal{N}(0,1)$ and $(v/\sigma_v) \sim \mathcal{N}(0,1)$. As a consequence, $q \to \beta z$ when $\sigma_\xi \to 0$ and $\sigma_v \to 0$, so that $\tilde{z} \to z$.

Now, consider the limits of the conditional expectations on z. Using Equation (19), $E_i(z) = E(q|z_i, \tilde{y}) = E(q|z_i, \tilde{z}) = \gamma z_i + \tilde{\gamma}\tilde{z}$, with $\gamma = \sigma_\varepsilon^{-2}/[\sigma_\varepsilon^{-2} + \sigma_z^{-2} + P]$, $\tilde{\gamma} = P/[\sigma_\varepsilon^{-2} + \sigma_z^{-2} + P]$, and $P = \kappa^2(\sigma_v^2 + \beta^2\sigma_\xi^2)^{-1}$. As $\varrho \to 1$, $P \to \beta^2(\sigma_v^2 + \beta^2\sigma_\xi^2)^{-1}$. When, additionally, $\sigma_\xi \to 0$ and $\sigma_v \to 0$, $P \to +\infty$, provided $\beta \neq 0$. As a result, $\gamma \to 0$, $\tilde{\gamma} \to 1$, and $E_i(z) \to \tilde{z}$, so that $E_i(z) \to z$.

According to Equation (23), the conditional expectation of q is $E_i(q) = E(q|z_i, \tilde{y}) = q + \lambda[E(z|z_i, \tilde{z}) - z]$. Since both λ and $E(z|z_i, \tilde{z}) - z$ converge to 0, we get that $E_i(q) \to q$.

Finally, as $\chi = 0$, Equation (13) holds, so that $p_i \to p^*$, as $E_i(z) \to z$ and $E_i(q) \to q$. The solution of the full model therefore converges to the one of the simple model when $\chi = 0$, $\varrho \to 1$, $\sigma_\xi \to 0$, $\sigma_v \to 0$ and $\beta \neq 0$. The simple model can thus be interpreted as a limit case of the full model.

B.8 Solution for λ_c

Lemma 1 *For a given policy parameter β, λ_c is characterized in equilibrium by*

$$\lambda_c = \frac{(\varrho - 1)(1 - \chi)\gamma_c\delta}{1 + [\varrho(1 - \chi) - 1]\gamma_c}, \tag{53}$$

with $\gamma_c = \sigma_\varepsilon^{-2}/(\sigma_z^{-2} + \sigma_\varepsilon^{-2} + P_c)$, where P_c is the combined precision of z^{cb} and $\bar{z}$.

As for Lemma 1, we first solve for the equilibrium pricing equation, given our guess of the endogenous signal: $\bar{z} = z - \lambda_c^{-1} v$, then characterize λ_c, and finally study existence and unicity of the solution.

Solving the pricing equation The price-setting equation (47) can be re-written as

$$p_i \;=\; \chi E_i(p) + (1-\chi) E_i[(\lambda_c - \delta) z + \beta z^{cb} - \lambda_c \bar{z}] \tag{54}$$

We make the following guess: $p_i = \alpha z_i + \bar{\alpha} \bar{z} + \alpha^{cb} z^{cb}$. This yields $p = \alpha z + \bar{\alpha} \bar{z} + \alpha^{cb} z^{cb}$. Taking expectations of p and z and replacing in (54), we get

$$\begin{array}{rcllll} p_i & = & \Big\{ \chi \alpha \gamma & +(1-\chi)(\lambda_c - \delta)\gamma & \Big\} z_i \\ & + & \Big\{ \chi \Big[\alpha \bar{\gamma} + \bar{\alpha} \Big] & +(1-\chi) \Big[(\lambda_c - \delta) \bar{\gamma} - \lambda_c \Big] & \Big\} \bar{z} \\ & + & \Big\{ \chi \Big[\alpha \gamma^{cb} + \alpha^{cb} \Big] & +(1-\chi) \Big[(\lambda_c - \delta) \gamma^{cb} + \beta \Big] & \Big\} z^{cb} \end{array}$$

where γ, $\bar{\gamma}$ and γ^{cb} are the Bayesian weights associated to z_i, $\bar{z}$ and z^{cb} respectively, so that $E_i(z) = \gamma z_i + \bar{\gamma} \bar{z} + \gamma^{cb} \bar{z}$. After identifying the coefficients and re-arranging we find

$$\alpha = \frac{(1-\chi)(\lambda_c - \delta)\gamma}{1 - \chi\gamma} \qquad \bar{\alpha} = -\lambda_c + \frac{(\lambda_c - \delta)\bar{\gamma}}{1 - \chi\gamma} \qquad \alpha^{cb} = \beta + \frac{(\lambda_c - \delta)\gamma^{cb}}{1 - \chi\gamma} \tag{55}$$

Characterization Firms observe the demand index: $\tilde{y} = q + (\varrho - 1) p = \beta z^{cb} + v + (\varrho - 1)(\alpha z + \bar{\alpha} \bar{z} + \alpha^{cb} z^{cb})$, where we have used $p = \alpha z + \bar{\alpha} \bar{y} + \alpha^{cb} z^{cb}$. $\tilde{y}$ can be a function of z^{cb} and $\bar{z}$ only. Therefore, the term $v + (\varrho - 1)\alpha z$ must be proportional to $\bar{z} = z - \lambda^{-1} v$.

Identifying the coefficients, we get

$$\lambda = -(\varrho - 1)\alpha$$

using the solution for α given by (55), this yields (53).

Existence and unicity Equation (53) can be rewritten as $x(\lambda_c) = \lambda_c$ with

$$x(\lambda_c) = \frac{(\varrho - 1)(1-\chi)\sigma_\varepsilon^{-2}\delta}{\sigma_z^{-2} + \lambda_c^2 \sigma_v^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}} \tag{56}$$

x is strictly positive so λ_c cannot be negative. On $\mathbb{R}+$, x is continuously decreasing with bounded values in $(0, \bar{\lambda}]$, $\bar{\lambda} = (\varrho - 1)(1-\chi)\sigma_\varepsilon^{-2}\delta / [\sigma_z^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}]$, so a unique solution λ_c to (53) exists.

B.9 Proof of Equations (26)

Since we can write $p^* = (\beta - \delta)z + \nu = \beta z^{cb} + (\lambda_c - \delta)z - \lambda_c \bar{z}$, we have

$$p - p^* = \frac{(\lambda_c - \delta)}{1 - \chi\gamma_c}[(1 - \gamma_c - \bar{\gamma}_c - \gamma_c^{cb})z - \bar{\gamma}_c\lambda_c^{-1}v - \gamma_c^{cb}\xi]$$

Using the above expression for p_i, we can also write

$$p_i - p = \frac{(1-\chi)(\lambda_c - \delta)}{1 - \chi\gamma_c}\gamma_c\varepsilon_i$$

Finally, note that

$$\begin{aligned} \bar{E}z - z &= (1 - \gamma_c - \bar{\gamma}_c - \gamma_c^{cb})z - \bar{\gamma}_c\lambda_c^{-1}v - \gamma_c^{cb}\xi \\ E_i z - \bar{E}z &= \gamma_c\varepsilon_i \end{aligned}$$

which yields (26).

B.10 Proof of Proposition 2

Here we prove that the precision under communication is strictly larger than in the absence of communication, except when $\beta = \beta^*$, in which case they are equal.

Suppose $P(\beta^*) = P_c$. This would imply:

$$\kappa(\beta^*)^2(\sigma_v^2 + (\beta^*)^2\sigma_\xi^2)^{-1} = \lambda_c^2\sigma_v^{-2} + \sigma_\xi^{-2} \tag{57}$$

Besides, according to (21) and (53), it would imply that:

$$\lambda_c = \lambda(\beta^*)$$

Using the characterization of β^* given by Equations (B.6) and (25), we know that

$$\begin{aligned} \kappa(\beta^*) &= \beta^* \tfrac{\sigma_v^2 + (\beta^*)^2\sigma_\xi^2}{(\beta^*)^2\sigma_\xi^2} \\ \lambda(\beta^*) &= \tfrac{\beta^*\sigma_v^2}{(\beta^*)^2\sigma_\xi^2} \end{aligned}$$

Replacing λ_c and κ in (57), then replacing λ, we get

$$\begin{aligned} \tfrac{(\beta^*)^2\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^2\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^{-1}}{[(\beta^*)^2\sigma_\xi^2]^2} &= \tfrac{(\beta^*)^2\sigma_v^2}{[(\beta^*)^2\sigma_\xi^2]^2} + \sigma_\xi^{-2} \\ \tfrac{\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^2\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]^{-1}}{(\beta^*)^2\sigma_\xi^4} &= \tfrac{\sigma_v^4\sigma_v^{-2}}{(\beta^*)^2\sigma_\xi^4} + \tfrac{(\beta^*)^2\sigma_\xi^2}{(\beta^*)^2\sigma_\xi^4} \\ \tfrac{\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]}{(\beta^*)^2\sigma_\xi^4} &= \tfrac{\left[\sigma_v^2+(\beta^*)^2\sigma_\xi^2\right]}{(\beta^*)^2\sigma_\xi^4} \end{aligned}$$

which is always true.

Therefore, $P(\beta^*) = P_c$. Since $P(\beta^*) > P(\beta)$ for all $\beta \neq \beta^*$, then $P(\beta) < P_c$ for all $\beta \neq \beta^*$.

B.11 Weight of z^{cb} in the pricing equation

We consider the case when $\beta = \beta^*$. Note that in that case $\kappa_c = \kappa$, $\lambda_c = \lambda$, $\gamma_c = \gamma$ and $\tilde{z}_c = \tilde{z}$. We therefore discard in what follows the subscript c.

Consider now the optimal pricing equation $p_i = \alpha z_i + \bar{\alpha}\bar{z} + \alpha^{cb} z^{cb}$, where α, $\bar{\alpha}$,α^{cb}, are defined in (55). We now show that $\bar{\alpha}\bar{z} + \alpha^{cb} z^{cb} = \tilde{\alpha}\tilde{z}$, where $\tilde{\alpha}$ is given by (48).

We begin by writing

$$\begin{aligned}
\bar{\alpha}\bar{z} + \alpha^{cb} z^{cb} &= \left[\kappa + \tfrac{(\lambda-\delta)(\bar{\gamma}+\gamma^{cb})}{1-\chi\gamma}\right] z \\
& \quad - \left[\lambda + \tfrac{(\lambda-\delta)\bar{\gamma}}{1-\chi\gamma}\right] \lambda^{-1} v \\
&= \left[\beta + \tfrac{(\lambda-\delta)\gamma^{cb}}{1-\chi\gamma}\right] \xi
\end{aligned}$$

From Equation (21) in Proposition 1, we have at the optimum

$$\frac{-\lambda}{\beta} = \frac{\bar{\gamma}}{\gamma^{cb}}$$

Hence

$$\frac{\kappa}{\beta} = \frac{\bar{\gamma}+\gamma^{cb}}{\gamma^{cb}} \qquad \frac{\kappa}{-\lambda} = \frac{\bar{\gamma}+\gamma^{cb}}{\bar{\gamma}}$$

Replacing β and $-\lambda$ using these expressions and rearranging, we obtain:

$$\begin{aligned}
\bar{\alpha}\bar{z} + \alpha^{cb} z^{cb} &= \left[\kappa + \tfrac{(\lambda-\delta)(\bar{\gamma}+\gamma^{cb})}{1-\chi\gamma}\right] \left[z + \kappa^{-1}(v + \beta\xi)\right] \\
&= \tilde{\alpha}\tilde{z}
\end{aligned}$$

This means that, once we use the endogenous signal $\tilde{z}$, the weight of z^{cb} in the optimal pricing equation is zero.

B.12 Solving the model with a price target

Here we sketch the proofs, as they follow the same steps as for the baseline case. We first solve for the equilibrium pricing equation, given our guess of the endogenous signal: $\tilde{z}_p = z + \kappa_p^{-1}\nu_p$, with $\nu_p = \beta_p \xi_p + v$, then characterize κ_p. Finally, we solve for the optimal β_p.

Solving the pricing equation The price-setting equation (47) can be re-written as

$$p_i = \chi E_i(p) + (1-\chi) E_i[\kappa_p \tilde{z}_p - \beta_p p - \kappa_p z - \delta z] \tag{58}$$

We make the following guess: $p_i = \alpha z_i + \tilde{\alpha}_p \tilde{z}_p$. This yields $p = \alpha z + \tilde{\alpha}_p \tilde{z}_p$. Taking expectations of p and z and replacing in (58), then after some calculations, we can identify the coefficients and obtain after rearranging:

$$\alpha = \frac{-(1-\chi)(\kappa_p + \delta)\gamma}{(1+\beta_p)[1 - \chi\gamma + (1-\chi)\gamma\beta_p]} \qquad \tilde{\alpha}_p = \frac{\kappa_p}{1+\beta_p} - \frac{(\kappa_p + \delta)\tilde{\gamma}}{(1+\beta_p)[1 - \chi\gamma + (1-\chi)\gamma\beta_p]} \tag{59}$$

Characterization of κ_p Firms observe the demand index: $\tilde{y} = q + (\varrho - 1)p = (-\beta_p + \varrho - 1)p + \kappa_p \kappa_p^{-1} \nu_p$. Using our guess-and-verify solution for p_i, we obtain p and then replace in $\tilde{y}$, which yields:

$$\tilde{y} = (-\beta_p + \varrho - 1)(\alpha z + \tilde{\alpha}_p \tilde{z}_p) + \kappa_p \kappa_p^{-1} \nu_p$$

$\tilde{y}$ depends linearly on z, v and $\tilde{z}_p$. For our guess to be true, we need that $\tilde{y} - (-\beta_p + \varrho - 1)\tilde{\alpha}_p \tilde{z}_p = f\bar{z}$ for some constant f. Identifying the coefficients, we get

$$(-\beta_p + \varrho - 1)\alpha = \kappa_p$$

Replacing α and rearranging, we obtain

$$\kappa_p = \frac{-(1-\chi)[\varrho - 1 - \beta_p]\gamma\delta}{1 + \gamma[\varrho(1-\chi) - 1]}$$

which yields (28).

Optimal β_p Our proof follows the lines of the proof of lemma 2 and Proposition 1. The steps are very similar because the optimal β_p maximizes the precision of $\tilde{z}_p$, $P_p(\beta_p) = \kappa_p^2 [\beta_p^2 \sigma_\xi^2 + \sigma_v^2]^{-1}$, which is of the same form as P.

We first determine the price gaps. Since we can write $p = \alpha z + \tilde{\alpha}_p \tilde{z}_p$ and $p^* = \kappa_p \tilde{z}_p - \beta_p p - (\kappa_p + \delta)z = (\kappa_p - \beta_p \tilde{\alpha}_p)\tilde{z}_p - (\kappa_p + \beta_p \alpha + \delta)z$, we get

$$p - p^* = \frac{\kappa_p + \delta}{1 - \chi\gamma + (1-\chi)\gamma\beta}[(1 - \gamma - \tilde{\gamma})z - \tilde{\gamma}\kappa_p^{-1}\nu]$$

Using the above expression for p_i, we can also write

$$p_i - p = \frac{-(1-\chi)(\kappa_p + \delta)}{1 - \chi\gamma + (1-\chi)\gamma\beta}\gamma\varepsilon_i$$

We can then write more explicitly the loss function:

$$L = \delta^2 \frac{\sigma_z^{-2} + P_p(\beta_p) + \Phi(1-\chi)^2\sigma_\varepsilon^{-2}}{[\sigma_z^{-2} + P_p(\beta_p) + \varrho(1-\chi)\sigma_\varepsilon^{-2}]^2} \tag{60}$$

This expression is obtained by replacing the above price gap and the individual deviations in the loss function, then replacing κ_p using (28), and finally by replacing γ and $\tilde{\gamma}$ and by their expressions.

The expression for the loss function is exactly identical to (50), suggesting that, as in the baseline, the optimal β_p maximizes the precision of the endogenous signal P_p.

As P_p has the same expression as P, the optimal β_p, denoted β_p^*, must satisfy the same optimality condition (52). However, now the expression for κ_p, (28), is different from (20), yielding

$$\kappa_p'(\beta_p^*) = \frac{(1-\chi)\gamma\delta\beta_p}{1+\gamma[\varrho(1-\chi)-1]}$$

where we use the fact that $\partial log(P_p)(\beta_p)/\partial\beta_p = 0$ for $\beta_p = \beta_p^*$.

Replacing in $\partial\log(P)/\partial\beta$ and rearranging, we find that β_p^* must satisfy Equation (29), which characterizes β_p^* uniquely.

B.13 Solving the dynamic model

Combining the expression for nominal demand (33) along with the endogenous signal $\tilde{y} = q + (\varrho - 1)p$, we determine that the endogenous signal can be written as follows

$$\tilde{y}_t = \frac{1}{\phi}(\beta z_t + \nu_t) + \left(\varrho - \frac{1}{\phi}\right)p_t$$

rescaling $\tilde{y}$, we get

$$\phi\tilde{y}_t = \beta z_t + \nu_t + (\phi\varrho - 1)p_t$$

$\phi\tilde{y}_t$ is exactly of the same form as $\tilde{y}$ in the static case, except that ϱ has been replaced with $\phi\varrho$. The solutions and optimal policy are then adjusted accordingly.

B.14 Solving the model with mark-up shocks

We proceed as in the baseline case to derive the average price gap and the individual deviations from the mean:

$$\begin{array}{rclccc} p_{-}p^* & = & -(\lambda_\rho - \delta_\rho) & (1+\gamma_\rho\tilde{\chi}_\rho) & \left[\bar{E}(\rho) - \rho\right] + \delta_\rho\rho \\ p_i - p & = & -(\lambda_\rho - \delta_\rho) & [1-(1-\gamma_\rho)\tilde{\chi}_\rho] & \left[E_i(\rho) - \bar{E}(\rho)\right] \end{array} \tag{61}$$

where $\tilde{\chi}_\rho = \chi/(1-\chi\gamma_\rho)$, γ_ρ and $\tilde{\gamma}_\rho$ are defined as in (**??**), $E_i(\rho)$ follows (19) and $\bar{E}(\rho) = \int_0^1 E_i(\rho)di$. λ_ρ is

$$\lambda_\rho = \frac{-(\varrho-1)(1-\chi)\delta_\rho\gamma_\rho}{1+[\varrho(1-\chi)-1]\gamma_\rho} \tag{62}$$

and $\kappa_\rho = \beta_\rho + \lambda_\rho$.

To determine optimal policy, we proceed as for the proof of Lemma 2 and write more explicitly the loss function:

$$L = \delta_\rho^2 \frac{[P_\rho(\beta_\rho) + \varrho(1-\chi)\sigma_\omega^{-2}]^2\sigma_\rho^2 + P_\rho(\beta_\rho) + \Phi(1-\chi)^2\sigma_\omega^{-2}}{\left[\sigma_\rho^{-2} + P_\rho(\beta_\rho) + \rho(1-\chi)\sigma_\omega^{-2}\right]^2} \tag{63}$$

where $P_\rho(\beta_\rho) = \kappa_\rho(\beta_\rho)^2(\sigma_v^2 + \beta_\rho^2\sigma_{\xi_\rho}^2)^{-1}$. This expression is obtained by replacing the output gap and the individual deviations using (61), then replacing λ_ρ using (62) and finally by replacing γ_ρ, $\tilde{\gamma}_\rho$ and $\tilde{\chi}_\rho$ by their expressions.

Replacing $\Phi = \varrho/(1-\chi)$, we obtain

$$\begin{aligned} L &= \delta_\rho^2 \tfrac{[P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}]^2\sigma_\rho^2+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}}{\left[\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}\right]^2} \\ &= \delta_\rho^2\sigma_\rho^2[P_\rho(\beta_\rho) + \varrho(1-\chi)\sigma_\omega^{-2}]\tfrac{\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}}{\left[\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}\right]^2} \\ &= \delta_\rho^2\sigma_\rho^2\tfrac{P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}}{\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_\omega^{-2}} \end{aligned} \tag{64}$$

L depends on β_ρ only through the precision $P_\rho(\beta_\rho)$. L increases monotonously with the precision $P_\rho(\beta_\rho)$. Therefore, the level of β_ρ that minimizes the precision also minimizes L. That level is such that $\kappa_\rho = 0$, which implies $\beta_\rho = -\lambda_\rho$. This yields expression (38)

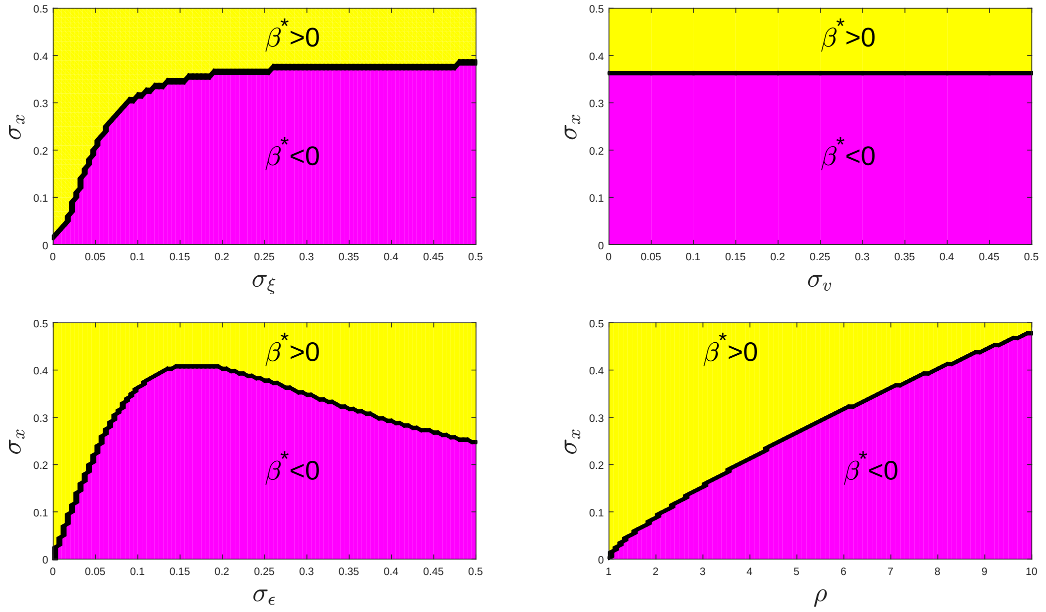
C Additional results

C.1 More on the role of parameters

In this exercise we investigate further the range of parameters where the signalling channel dominates, with a focus on the endogenous signal's precision. Figure 6 shows the parameter spaces where β^* is positive (dominant surprise channel) and negative (dominant signalling channel).

As stressed in the paper, it appears that the optimal β is typically positive for large values of σ_x and typically negative for small values, for given σ_ξ, σ_v, σ_ϵ or

Figure 6: The role of parameters for the sign of β^*



Note: As default values, we set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.2$.

ρ. The threshold σ_x above which β^* turns positive goes to zero only in limit case (when ρ or σ_ϵ go to zero). By looking at this threshold, we can determine how the parameters affect the relative strength of the signalling and surprise channels.

The threshold σ_x, above which β^* turns positive, is increasing in σ_ξ, as shown in panel a). As σ_ξ gets larger, β^* can turn from positive (dominant surprise channel) to negative (dominant signalling channel) for a given σ_x. Because firms have private information on z and can put in perspective their demand signal with their information on z, a relatively less well informed central bank will tend to rely more on the signalling channel, rather than use the surprise channel. The signalling channel dominates for larger values of σ_x when the central bank has relatively poor information (σ_ξ is large).

As shown in panel b), threshold σ_x, above which β^* turns positive, is independent of σ_v. Monetary shocks do not affect the relative strength of the two channels.

Panel c) d) show respectively the role of σ_ϵ and ρ. Both a higher ρ and a lower σ_ϵ (a larger γ) make the demand signal a better "natural" signal of z (see

Lemma 1 in the paper), limiting the need for both a signalling policy and a price-stabilizing policy. So it is not clear how changing these parameters should affect the relative importance of the signalling and the surprise channels. The simulations show that the signalling channel dominates for larger values of σ_x when σ_ϵ is at an intermediate level, and when ρ is large.

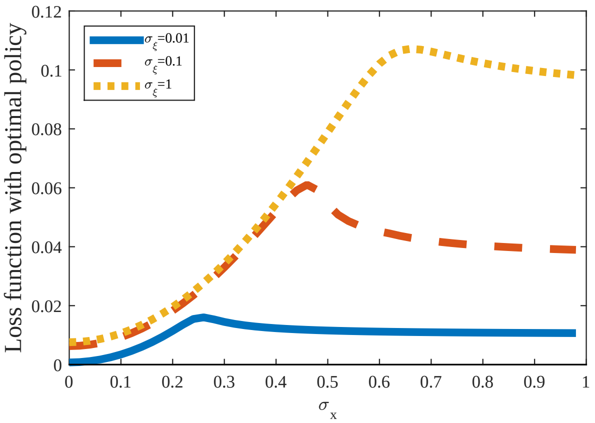
C.2 The role of endogenous attention

Rational inattention models consider contexts where σ_x is endogenous, because attention is costly. It is likely that, with endogenous attention, a price-stabilizing policy would reduce the benefits of attention, thus bringing σ_x to infinity, while a signalling policy would increase the benefits of attention, thus bringing σ_x to zero as it relies on volatile prices. It is then legitimate to ask whether price stabilization would not maximize welfare by limiting the need for firms to acquire information and hence minimizing the attention costs. This would potentially rehabilitate the surprise channel. This question is beyond the scope of the paper, but Figure 7 can help us get a sense of how this assumption could modify our results. It shows the loss function with the optimal policy ($\beta = \beta^*$), as a function of σ_x, for different values of σ_ξ. As shown earlier, low σ_x correspond to a negative β^* and volatile prices, while high σ_x correspond to a negative β^* and stable prices.

Note that the loss function is larger for intermediate σ_x, where neither the signalling channel nor the surprise channel is really effective. But the loss function is minimized when σ_x goes to zero, that is, when the signalling channel is at its best. As σ_x goes to infinity, the loss function declines as compared to intermediate values, but remains above its minimum. When the surprise channel is at its best, it does not dominate the signalling channel at its best. This is because the central bank has imperfect information, so it cannot achieve the first-best with the surprise policy. In this context, the optimum is reached when firms have a better information and the central bank exploits their attention to demand by transmitting its own information.

However, Figure 7 does not take into account the costs of attention. If attention to demand is very costly, then a higher welfare would be reached with a price-stabilizing policy. But if attention is not too costly, then the signalling policy would still be optimal.

Figure 7: Attention to demand, central bank information and the loss function



Note: We set $\varrho = 7$, $\phi + \eta = 0.5$, which yields $\delta = 2$ and $\chi = 0.5$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$.